# Density Functional Study on Geometrical Features and Electronic Structures of Di- $\mu$-oxo-Bridged $\left[\mathrm{Mn}_{2} \mathrm{O}_{\mathbf{2}}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ with $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}(\mathrm{IV})$ 

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#### Abstract

We report the geometrical features and electronic structures of di- $\mu$-oxo-bridged $\mathrm{Mn}-\mathrm{Mn}$ binuclear complexes with $\mathrm{H}_{2} \mathrm{O}$ ligands $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the iso- and mixed-valence oxidation states. All of the combinations among $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}(\mathrm{IV})$ ions are considered the oxidation states of the $\mathrm{Mn}-\mathrm{Mn}$ center, and the changes in molecular structure induced by the different electron configurations of Mn-based orbitals are investigated in relation to the oxygen-evolving complex (OEC) of photosystem II. The stable geometries of complexes are determined by using the hybrid-type density functional theory for both of the highest- and lowest-spin couplings between Mn sites, and the lowest-spin-coupled states are energetically more favorable than the highest-spin-coupled states except in the case of the complexes with the $\mathrm{Mn}(\mathrm{II})$ ion. The coordination positions of $\mathrm{H}_{2} \mathrm{O}$ ligands at the Mn (II) site tend to shift from the octahedral positions in contrast to those at the $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$ sites. The shape of the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core and the distances between the Mn ions and the $\mathrm{H}_{2} \mathrm{O}$ ligands vary depending on the electron occupations of the octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals on the Mn site with an antibonding nature for the $\mathrm{Mn}-$ ligand interactions, indicating the trend as $\mathrm{Mn}(\mathrm{II})-\mathrm{O}>\mathrm{Mn}(\mathrm{III})-\mathrm{O}$ and $\mathrm{Mn}-$ (IV) $-\mathrm{O}, \mathrm{O}-\mathrm{Mn}(\mathrm{II})-\mathrm{O}>\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}>\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ among the iso-valence $\mathrm{Mn}_{2} \mathrm{O}_{2}$ cores, and $\mathrm{O}-\mathrm{Mn}-$ (lower) $-\mathrm{O}<\mathrm{O}-\mathrm{Mn}$ (higher) -O within the mixed-valence $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core, and as $\mathrm{Mn}(\mathrm{II})-\mathrm{OH}_{2}$ and $\mathrm{Mn}(\mathrm{III})-$ $\mathrm{OH}_{2}>\mathrm{Mn}(\mathrm{IV})-\mathrm{OH}_{2}$ for the axial $\mathrm{H}_{2} \mathrm{O}$ ligand. The optimized geometries of model complexes are compared with the X-ray structure of the OEC, and it is suggested that the cubanelike Mn cluster of the active site may not contain a $\mathrm{Mn}(\mathrm{II})$ ion. The effective exchange integrals are estimated by applying the approximate spin projection to clarify the magnetic coupling between Mn sites, and the superexchange pathways through the di- $\mu$-oxo bridge are examined on the basis of the singly occupied magnetic orbitals derived from the singletcoupled natural orbitals in the broken-symmetry state. The comparisons of the calculated results between $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in this study and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ reported by McGrady et al. suggest that the symmetric pathways are dominant to the exchange coupling constant, and the crossed pathway would be less important for the former than it would for the latter in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ oxidation states.


## I. Introduction

Photosynthetic water oxidation in photosystem II (PSII) takes place at the tetranuclear Mn cluster as an active site, and the functional unit including the Mn cluster is referred to as the oxygen-evolving complex (OEC) or water-oxidizing complex (WOC). The oxidation reaction from water to dioxygen molecules is catalyzed by the OEC as $2 \mathrm{H}_{2} \mathrm{O} \rightarrow \mathrm{O}_{2}+4 \mathrm{e}^{-}+$ $4 \mathrm{H}^{+}$. Several models for the reaction mechanism of water oxidation or for the molecular structure of the OEC have been proposed by experimental and theoretical studies, and many related articles have been published (e.g., refs $1-16$ ).

The reaction cycle of water oxidation by the OEC has been known as the S-state catalytic cycle or the Kok cycle, in which the oxidation reaction proceeds through the five-step process from the $S_{0}$ state to the $S_{4}$ state. Figure 1 shows the $S$-state catalytic cycle schematically. ${ }^{1}$ The tetranuclear Mn cluster of the active site has been considered to be oxidized stepwise with releasing the electrons and protons through each of the oxidation

[^0]

Figure 1. S-state catalytic cycle of water oxidation by the OEC.
steps as indicated in Figure 1, then molecular dioxygen would be produced in the last step from the $\mathrm{S}_{4}$ state to the $\mathrm{S}_{0}$ state. The oxidation states of the four Mn ions at the reaction center in each of the $S$ states have been suggested, for example, as $\mathrm{Mn}_{4}$ (II, III, IV, IV) in the $\mathrm{S}_{0}$ state $\rightarrow \mathrm{Mn}_{4}$ (III,III,IV,IV) in the $\mathrm{S}_{1}$ state $\rightarrow \mathrm{Mn}_{4}$ (III,IV,IV,IV) in the $\mathrm{S}_{2}$ state $\rightarrow \mathrm{Mn}_{4^{-}}$ (III,IV,IV,IV) with the $\mu$-oxo radical in the $\mathrm{S}_{3}$ state $\rightarrow \mathrm{Mn}_{4^{-}}$ (III,IV,IV,IV) in the $\mathrm{S}_{4}$ state, ${ }^{1}$ and the iso- and mixed-valence $\mathrm{Mn}-\mathrm{Mn}$ pairs would appear in the S-state catalytic cycle. The
extended X-ray absorption fine structure (EXAFS) and electron spin-echo envelope modulation (ESEEM) spectroscopy have indicated that the first-shell ligands of the Mn cluster are mostly oxygens, one or two nitrogens, and possibly one chlorine. The dimer complex of di- $\mu$-oxo-bridged $\mathrm{Mn}-\mathrm{Mn}$ binuclear centers linked to each other by a mono- $\mu$-oxo bridge has been proposed as the working model for the tetranuclear Mn cluster in the OEC on the basis of the available experimental results. ${ }^{1}$

X-ray crystallographic studies of PSII have been reported in recent years. ${ }^{17-21}$ Although the 3D coordinates obtained by several experiments are available from the Protein Data Bank (PDB), the observed structures of the Mn cluster in the OEC are different from each other. The crystal structures from Synechococcus elongatus at $3.8 \AA$ resolution, ${ }^{17}$ Thermosynechococcus vulcanus at $3.7 \AA$ resolution, ${ }^{18}$ and Thermosynechococcus elongatus at $3.2 \AA$ resolution ${ }^{20}$ suggest a Y-shaped arrangement of the Mn ions, in which any Ca ions were not detected ${ }^{17,18}$ or only four metal ions were accommodated. ${ }^{20}$ However, the crystal structure from Thermosynechococcus elongatus at $3.5 \AA$ resolution ${ }^{19}$ strongly indicated the cubanelike arrangement of Mn and Ca ions with $\mu$-oxo bridges, such as $\mathrm{Mn}_{3} \mathrm{CaO}_{4}$, in which the details of the surrounding coordination sphere around the Mn cluster were also assigned. Thus, the cubanelike models for the reaction mechanism of water oxidation by the OEC are now widely discussed. ${ }^{22-27}$ However, more recently, the X-ray structure of PSII from Thermosynechococcus elongates at $3.0 \AA$ Å resolution has been reported, ${ }^{21}$ in which the Mn cluster was best approximated by the hook-shaped arrangement of Mn ions resembling the Y-shaped structure. The position of the Ca ion was suggested as the vertex of a trigonal pyramid formed by the Mn1, Mn2, and Mn3 ions numbered from the highest electron-density, and the Mn4 ion was linked to the Mn3 ion. The $\mathrm{Mn}-\mathrm{Mn}$ distances were estimated to be about $2.7 \AA$ for $\mathrm{Mn} 1-\mathrm{Mn} 2$ and $\mathrm{Mn} 2-\mathrm{Mn} 3$ and about $3.3 \AA$ for Mn1-Mn3 and Mn3-Mn4, and it was suggested that the $\mathrm{Mn}-\mathrm{Mn}$ pairs with shorter and longer separations are connected by di- $\mu$-oxo and mono- $\mu$-oxo bridges, respectively, although the positions of $\mu$-oxo ions were not reported. The $\mathrm{Mn}-\mathrm{Ca}$ distances forming the trigonal pyramid were indicated to be equidistant for the Mn1, Mn2, and Mn3 ions at about $3.4 \AA$.

Although several experimental structures of the Mn cluster in the OEC are available as described above, the detailed 3D geometry has not yet been determined crystallographically because of the low resolution of the X-ray structures. Therefore, computational investigations of geometrical features and electronic structures for model complexes are important to examine water oxidation by the OEC, and we studied the basic unit of the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ face constructing the cubanelike Mn cluster reported in ref 19 in this work. Figure 2 shows the observed X-ray structure of the cubanelike Mn cluster in the OEC (pdb code: 1S5L). Three Mn ions and one Ca ion form the cubanelike structure by tri- $\mu$-oxo bridges, and the outer Mn ion is linked to $\mathrm{Mn}_{3} \mathrm{CaO}_{4}$ in part by a mono- $\mu$-oxo bridge, in which the $\mathrm{Mn}-$ Mn distances were reported to be about $2.7 \AA$ within the cubanelike cluster and about $3.3 \AA$ between the cubanelike cluster and the outer ion, whereas the $\mathrm{Mn}-\mathrm{Ca}$ distance was reported to be about $3.4 \AA$.

In the field of computational chemistry, quantum chemical calculations have been carried out for several models or synthetic compounds of the Mn complex with $\mu$-oxo bridges in relation to the Mn cluster in the OEC. ${ }^{28-38}$ Regarding the studies of the di- $\mu$-oxo-bridged Mn dimer using the broken-symmetry DFT method, the magnetic interactions of $\left.\left[\mathrm{Mn}_{2} \mathrm{O}_{2} \text { (pic) }\right)_{4}\right]$ (picH $=$ picolinic acid) and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2} \mathrm{X}_{8}\right]^{q+}\left(\mathrm{X}=\mathrm{H}^{-}, \mathrm{F}^{-}, \mathrm{Cl}^{-}, \mathrm{H}_{2} \mathrm{O}\right.$, and


Figure 2. X-ray structure of the cubanelike Mn cluster in the OEC (pdb code: 1S5L). ${ }^{19}$ The atoms without an element symbol indicate carbon atoms.
$\mathrm{NH}_{3}$ ) were examined in ref 32, the redox-induced changes in molecular geometry and magnetic properties of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ were investigated in ref 30 , and the geometrical parameters and electronic structures of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{6}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}\right]^{q+}$ were reported in refs 34 and 37. Theoretical examinations of the cubanelike model using the X-ray structure were recently attempted for the oxidation states in the S -state catalytic cycle, ${ }^{39,40}$ and the spin structures with noncollinear spin alignment were predicted by applying the Hartree-Fock calculation using general spin orbitals (GSO-HF) in ref 40. However, the geometrical structures and electron configurations of the $\mathrm{S}_{0}-$ $\mathrm{S}_{4}$ states for the cubanelike Mn cluster in the OEC have not yet been clarified from a theoretical viewpoint.

In the previous paper, ${ }^{41}$ we investigated the geometrical and electronic structures of model complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}(q$ $=0,2,4)$ using the hybrid-type density functional theory (DFT) as the first step toward a theoretical examination of the reaction mechanism for water oxidation by the OEC. The stable geometries of energy minima were respectively determined for each of the highest- and lowest-spin states by applying the LanL2DZ ${ }^{42}$ effective core potential (ECP) to the Mn ions, and the relative stabilities, charge and spin distributions, and electron configurations in the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}-$ (IV) $-\mathrm{Mn}(\mathrm{IV})$ oxidation states were reported. However, the atomic charges on the oxidized Mn ions obtained from the ECP method seemed to be too small. Therefore, the model complexes should be re-examined using the all-electron DFT method.

In the present article, we study the geometrical characteristics of model complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}(q=0-4)$ and their changes depending on the oxidation state of the $\mathrm{Mn}-\mathrm{Mn}$ center by using the all-electron hybrid-type DFT calculation. The highest- and lowest-spin couplings between the Mn sites are considered in the $\mathrm{Mn}-\mathrm{Mn}$ center. The iso- and mixed-valence combinations among $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}(\mathrm{IV})$ ions, the isovalence centers of $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}-$ (IV) $-\mathrm{Mn}(\mathrm{IV})$ and the mixed-valence centers of $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$, $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$, and the relative stabilities of both spin alignments in each oxidation state are examined. The $\mathrm{Mn}_{2} \mathrm{O}_{2}$ geometries of the optimized complexes are com-


Figure 3. Model complex with a di- $\mu$-oxo bridged $\mathrm{Mn}-\mathrm{Mn}$ binuclear center and numbering of $\mathrm{Mn}_{2} \mathrm{O}_{2}$ atoms and $\mathrm{H}_{2} \mathrm{O}$ molecules.
pared with the X-ray structure of the cubanelike $\mathrm{Mn}_{3} \mathrm{CaO}_{4}$ cluster in the OEC. The distributions of charge and spin densities in the complexes are evaluated using the Mulliken population analysis, and the singly occupied electron configurations of the $\mathrm{Mn}-\mathrm{Mn}$ centers are elucidated using the natural orbital analysis. The magnetic interactions between Mn sites are estimated in terms of the effective exchange integrals and superexchange pathways mediated by the $\mu$-oxo bridge. The calculated structures, electron populations, and magnetic properties of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in this study are compared with those of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ given in ref 30 .

## II. Model Complex and Computational Method

Because the di- $\mu$-oxo-bridged $\mathrm{Mn}-\mathrm{Mn}$ center forms the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ faces of the cubanelike Mn cluster in the OEC as shown in Figure 2 and it has been considered that only two of the four Mn sites may be sensitive to a photooxidation reaction of water molecules in many of earlier models, we study the $\mathrm{Mn}-\mathrm{Mn}$ binuclear complexes bridged by di- $\mu$-oxo ions $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ shown in Figure 3 as a simplified model of the tetranuclear Mn cluster in the OEC. Eight $\mathrm{H}_{2} \mathrm{O}$ molecules are placed around Mn ions as ligands to form the six-coordinated Mn sites. We respectively take the $\mathrm{Mn}-\mathrm{Mn}$ and $\mathrm{O}-\mathrm{O}$ axes as the $y$ and $x$ axes, and the $z$ axis is perpendicular to the plane formed by the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core facing the axial direction. Therefore, two octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals at each Mn site include the $x y$ (equatorial) and $z^{2}$ (axial) components of the 3d orbital, respectively. In this article, we apply the following numbering of atoms and molecules to the model complexes: the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ atoms, axial $\mathrm{H}_{2} \mathrm{O}$ molecules, and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules are respectively numbered from 1 to 4 , from 5 to 8, and from 9 to 12 as indicated in Figure 3. It is noted that the Mn1 and Mn2 ions, respectively, correspond to the Mn ions in lower and higher oxidation states of a mixedvalence $\mathrm{Mn}-\mathrm{Mn}$ pair in the case of mixed-valence complexes.

In the S-state catalytic cycle, the iso- and mixed-valence states of $\mathrm{Mn}-\mathrm{Mn}$ pairs are generated through the oxidation steps. We consider the possible iso- and mixed-valence combinations of $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}$ (III), and $\mathrm{Mn}(\mathrm{IV})$ ions as the $\mathrm{Mn}-\mathrm{Mn}$ center of complexes as follows: $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (II) (1), Mn (III) -Mn (III) (2), and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ (3) for the iso-valence complexes; Mn(II) $-\mathrm{Mn}(\mathrm{III})$ (4), $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ (5), and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ (6) for the mixed-valence complexes. The formal charges of the Mn and $\mu$-oxo ions are respectively taken as $2+$ for $\mathrm{Mn}(\mathrm{II})$, $3+$ for $\mathrm{Mn}(\mathrm{III})$, and $4+$ for $\mathrm{Mn}(\mathrm{IV})$ and as $2-$ for $\mu$-oxo, by which the total charge $q$ of each complex is given as 0 for $\mathbf{1}, 1$ for $\mathbf{4}, 2$ for $\mathbf{2}$ and 5, 3 for $\mathbf{6}$, and 4 for $\mathbf{3}$. The highest- and lowest-spin couplings between locally parallel-spin alignments at each of the two Mn ions are applied to specify the electronic states of the complexes. The formal 3d electron occupations of Mn ions are thus open shell with the parallel spins as $3 \mathrm{~d}^{5}$ for $\mathrm{Mn}(\mathrm{II}), 3 \mathrm{~d}^{4}$ for $\mathrm{Mn}(\mathrm{III})$, and $3 \mathrm{~d}^{3}$ for $\mathrm{Mn}(\mathrm{IV})$, and the locally parallel spins on the $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}(\mathrm{IV})$ centers are coupled with the same and opposite directions between two Mn sites, respectively, in the highest- and lowest-spin configurations.

These spin couplings lead to the total spin $S$ of $10 / 2$ for $\mathbf{1}, 9 / 2$ for $\mathbf{4}, 8 / 2$ for $\mathbf{2}$ and $\mathbf{5}, 7 / 2$ for $\mathbf{6}$, and $6 / 2$ for $\mathbf{3}$ in the highest-spin configurations, whereas the total spin $S$ is $0 / 2$ for $\mathbf{1 - 3}, 1 / 2$ for $\mathbf{4}$ and $\mathbf{6}$, and $\frac{2}{2}$ for $\mathbf{5}$ in the lowest-spin configurations.

The unrestricted hybrid-type DFT method using the B3LYP combination of exchange and correlation functionals (UB3LYP) ${ }^{43,44}$ was applied to the full geometry optimization of model complexes under no spatial symmetry constraints. The optimized stationary points were confirmed to be or not to be energy minimums using the vibrational frequency analysis. The Ahlrichs VDZ, ${ }^{45}$ Pople 6-31G(d), ${ }^{46}$ and Pople $6-31 \mathrm{G}^{46}$ sets were employed as the basis functions of the $\mathrm{Mn}, \mathrm{O}$, and H species, respectively. The lowest-spin configurations were obtained as broken-symmetry solutions. The Mulliken population analysis was used to evaluate the distributions of charge and spin densities. All the results given in this article were calculated using the Gaussian 98 program package. ${ }^{47}$

## III. Effective Exchange Integral and Magnetic Orbital

The effective exchange integral represents the magnitude of magnetic interactions between spin sites, and the positive or negative value respectively corresponds to the parallel (ferromagnetic) or antiparallel (antiferromagnetic) spin coupling. Several approximation schemes to calculate the effective exchange integral by employing the broken-symmetry approach have been proposed. ${ }^{48-51}$

The Heisenberg spin Hamiltonian is used for the iso-valence binuclear complexes with localized spins,

$$
\begin{equation*}
\hat{H}=-2 J \overrightarrow{\mathrm{~S}}_{a} \cdot \overrightarrow{\mathrm{~S}}_{b} \tag{1}
\end{equation*}
$$

where $J$ is the effective exchange integral (exchange coupling constant) between the $a$ th and $b$ th metal sites with spin vectors $\vec{S}_{a}$ and $\vec{S}_{b}$. Three-types of approximation are applicable to obtain the $J$ value, ${ }^{48-50}$

$$
\begin{gather*}
J^{(1)}=\frac{{ }^{\mathrm{LS}} E-{ }^{\mathrm{HS}} E}{S_{\mathrm{max}}{ }^{2}}  \tag{2}\\
J^{(2)}=\frac{{ }^{\mathrm{LS}} E-{ }^{\mathrm{HS}} E}{S_{\max }\left(S_{\max }+1\right)}  \tag{3}\\
J^{(3)}=\frac{{ }^{\mathrm{LS}} E-{ }^{\mathrm{HS}} E}{{ }^{\mathrm{HS}}\left\langle\hat{S}^{2}\right\rangle-{ }^{\mathrm{LS}}\left\langle\hat{S}^{2}\right\rangle} \tag{4}
\end{gather*}
$$

where ${ }^{\mathrm{LS}} E$ and ${ }^{\mathrm{HS}} E$ represent the total energies of the low- and high-spin states, respectively, with the expectation values of total spin angular momentum ${ }^{\mathrm{LS}}\left\langle\hat{S}^{2}\right\rangle$ and ${ }^{\mathrm{HS}}\left\langle\hat{S}^{2}\right\rangle$.
A more general spin Hamiltonian including the resonance delocalization (double exchange) effects is used for the mixedvalence binuclear complexes,

$$
\begin{equation*}
\hat{H}=-2 J \overrightarrow{\mathrm{~S}}_{a} \cdot \overrightarrow{\mathrm{~S}}_{b} \pm B\left(S+\frac{1}{2}\right) \tag{5}
\end{equation*}
$$

where $B$ is the resonance delocalization parameter. ${ }^{52}$ The $J$ value is approximated by two schemes using the average energy of two delocalized high-spin states denoted as $g$ and $u$, in which an extra electron is placed in the delocalized orbitals with the in-phase and out-of-phase combinations of specific (unpaired) metal 3d orbitals, respectively, ${ }^{48,51}$

$$
\begin{gather*}
B=\frac{{ }^{\mathrm{HS}} E_{\mathrm{u}}-{ }^{\mathrm{HS}} E_{\mathrm{g}}}{2 S_{\mathrm{max}}+1}  \tag{6}\\
J^{(4)}=\frac{{ }^{\mathrm{LS}} E-{ }^{\mathrm{HS}} E_{\mathrm{av}}}{\left(S_{\mathrm{max}}+\frac{1}{2}\right)\left(S_{\mathrm{max}}-\frac{1}{2}\right)}  \tag{7}\\
J^{(5)}=\frac{{ }^{\mathrm{LS}} E-{ }^{\mathrm{HS}} E_{\mathrm{av}}}{{ }^{\mathrm{HS}}\left\langle\hat{S}^{2}\right\rangle_{\mathrm{av}}-{ }^{\mathrm{LS}}\left\langle\hat{S}^{2}\right\rangle}  \tag{8}\\
{ }^{\mathrm{HS}} E_{\mathrm{av}}=\frac{{ }^{\mathrm{HS}} E_{\mathrm{g}}+{ }^{\mathrm{HS}} E_{\mathrm{u}}}{2}  \tag{9}\\
{ }^{\mathrm{HS}}\left\langle\hat{S}^{2}\right\rangle_{\mathrm{av}}=\frac{{ }^{\mathrm{HS}}\left\langle\hat{S}^{2}\right\rangle_{\mathrm{g}}+{ }^{\mathrm{HS}}\left\langle\hat{S}^{2}\right\rangle_{\mathrm{u}}}{2} \tag{10}
\end{gather*}
$$

It is noted that eqs 4 and 8 are derived from the approximate spin projection method developed by Yamaguchi et al. ${ }^{49-51}$ Because we have confirmed that eqs $2-4$ or eqs 7 and 8 give similar $J$ values for $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{9+}$ in this study (the $|J|$ values obtained from eq 3 are somewhat smaller than those obtained from eq 2 or eq 4), only the calculated $J$ values of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ using eq 4 or eq 8 are reported in this article.

The natural orbitals and their electron occupation numbers are obtained by diagonalizing the first-order reduced density matrix as the eigenvectors and eigenvalues, respectively,

$$
\begin{equation*}
\rho\left(r, r^{\prime}\right)=\sum_{i} n_{i} \phi_{i}(r) \phi_{i}^{*}\left(r^{\prime}\right) \tag{11}
\end{equation*}
$$

The pair of natural orbitals $\phi_{\mathrm{b}}$ (bonding) and $\phi_{\mathrm{a}}$ (antibonding) forming a singlet biradical in the broken-symmetry state can be specified by their electron occupation numbers, which give 2.0 as the sum of $n_{\mathrm{b}}$ and $n_{\mathrm{a}}$. The natural orbitals delocalized over two spin sites are related to the magnetic orbitals localized around one of the spin sites as follows,

$$
\begin{equation*}
\psi^{ \pm}=\sqrt{\frac{n_{\mathrm{b}}}{2}} \phi_{\mathrm{b}} \pm \sqrt{\frac{n_{\mathrm{a}}}{2}} \phi_{\mathrm{a}} \tag{12}
\end{equation*}
$$

By using eq 12, it is possible to transform the symmetryadapted natural orbitals of a singlet biradical pair into the broken-symmetry magnetic orbitals after applying the plus and minus mixing with their electron occupation numbers as the weighting factors. In this article, the superexchange pathways between Mn sites through the di- $\mu$-oxo bridge in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{9+}$ are examined on the basis of the magnetic orbitals.

The electron population on atom X is given as follows under the Mulliken partitioning,

$$
\begin{gather*}
P^{\mathrm{X}}=\sum_{i} n_{i} \sum_{r}^{\text {on } \mathrm{X}}\left(\sum_{s}^{\text {all }} C_{r i} C_{s i} S_{r s}\right)=\sum_{i} n_{i} \sum_{r}^{\text {on } \mathrm{X}} P_{r i}=\sum_{i} P_{i}^{\mathrm{X}}  \tag{13}\\
\sum_{r}^{\text {all }} P_{r i}=\sum_{r}^{\text {all all }} \sum_{s} C_{r i} C_{s i} S_{r s}=\left\langle\psi_{i} \mid \psi_{i}\right\rangle=1 \tag{14}
\end{gather*}
$$

where $C_{r i}$ and $C_{s i}$ denote the expansion coefficients for the $r$ th and $s$ th basis functions in the $i$ th molecular orbital and $S_{r s}$ indicates the overlap integral between the $r$ th and $s$ th basis functions. The atomic population $P^{\mathrm{X}}$ is obtained as the sum of contributions from each molecular orbital $P_{i}^{\mathrm{X}}$, and $P_{r i}$ is the gross orbital population of the $r$ th basis function in the $i$ th molecular orbital, representing the contribution of the $r$ th basis

TABLE 1: Total and Relative Energies of Iso- and Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathbf{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$

| complex | $S$ | $\left\langle S^{2}\right\rangle$ | total energy <br> $(\mathrm{au})$ | relative energy <br> $(\mathrm{kcal} / \mathrm{mol})$ |
| :---: | ---: | ---: | :---: | :---: |
| $\mathbf{1}$ | $10 / 2$ | 30.00 | -3063.71090 | 0.16 |
|  | $0 / 2$ | 4.99 | -3063.71117 | 0.00 |
| $\mathbf{2}$ | $8 / 2$ | 20.09 | -3063.23663 | 7.06 |
|  | $0 / 2$ | 4.01 | -3063.24788 | 0.00 |
| $\mathbf{3}$ | $6 / 2$ | 12.19 | -3061.99682 | 2.78 |
|  | $0 / 2$ | 3.02 | -3062.00126 | 0.00 |
| $\mathbf{4 a}$ | $9 / 2$ | 24.80 | -3063.54666 | 0.63 |
|  | $1 / 2$ | 4.77 | -3063.54765 | 0.00 |
| $\mathbf{4 b}$ | $9 / 2$ | 24.80 | -3063.54624 | 0.89 |
|  | $1 / 2$ | 4.78 | -3063.54719 | 0.29 |
| $\mathbf{5}$ | $8 / 2$ | 20.05 | -3063.21126 | 0.09 |
|  | $2 / 2$ | 4.99 | -3063.21140 | 0.00 |
| $\mathbf{6}$ | $7 / 2$ | 15.88 | -3062.72489 | 5.65 |
|  | $1 / 2$ | 3.76 | -3062.73390 | 0.00 |

function to $P_{i}^{\mathrm{X}}$ or the weight of the $r$ th basis function for electron distribution in the $i$ th molecular orbital. In this article, we use the $P_{r i}$ values (multiplied by 100) as the composition of the molecular orbitals.

## IV. Relative Stability of Model Complex

We found several energy minima of model complexes with similar or different locations of $\mathrm{H}_{2} \mathrm{O}$ ligands in different orientations except in $\mathbf{1}$. The energy intervals from the most stable structure were obtained as follows for the lowest-spin configurations: 0.1 and $0.7 \mathrm{kcal} / \mathrm{mol}$ for $\mathbf{2}$ and $\mathbf{3} ; 0.3,4.0,6.6$, 7.7 , and $10.3 \mathrm{kcal} / \mathrm{mol}$ for $\mathbf{4} ; 0.9,2.5$, and $5.5 \mathrm{kcal} / \mathrm{mol}$ for $\mathbf{5}$; $1.9 \mathrm{kcal} / \mathrm{mol}$ for $\mathbf{6}$. The relative stabilities among energy minima of iso-valence complexes 2 and $\mathbf{3}$ with similar $\mathrm{H}_{2} \mathrm{O}$ positions are almost equivalent (within $1 \mathrm{kcal} / \mathrm{mol}$ ), whereas some of the energy minima for mixed-valence complexes $\mathbf{4 - 6}$ with different $\mathrm{H}_{2} \mathrm{O}$ positons indicate a relatively large instability (more than $5 \mathrm{kcal} / \mathrm{mol}$ ). Hereafter in this article, we focus our attention on the most-stable energy minima of complexes $\mathbf{1}-\mathbf{6}$, and complex 4 has two most-stable energy minima with different coordination patterns of $\mathrm{H}_{2} \mathrm{O}$ ligand.

It is noted that the structures of the ECP energy minima for 2 that were reported in the previous study ${ }^{41}$ became the thirdorder saddle points with three imaginary frequencies by the allelectron calculations as follows: $142.3 i, 102.1 i$, and $84.6 i \mathrm{~cm}^{-1}$ (highest-spin configuration); $152.6 i, 110.6 i$, and $94.6 i \mathrm{~cm}^{-1}$ (lowest-spin configuration). The all-electron method predicted three energy minima with geometries that differ from the ECP geometries. However, energy minima with similar structures were obtained by the ECP and all-electron methods for $\mathbf{1}$ and 3.

We summarize the total and relative energies of iso- and mixed-valence complexes in Table 1 together with the expectation values of total spin angular momentum. The different energy minima of $\mathbf{4}$ with near energies are distinguished by alphabetical symbols as $\mathbf{4 a}$ and $\mathbf{4 b}$.

The expectation values of total spin angular momentum $\left\langle\hat{S}^{2}\right\rangle$ are obtained as 30.00 for $\mathbf{1}$ with $S=10 / 2,20.09$ for $\mathbf{2}$ with $S=$ $8 / 2,12.19$ for $\mathbf{3}$ with $S=6 / 2,24.80$ for $\mathbf{4 a}$ and $\mathbf{4 b}$ with $S=9 / 2$, 20.05 for $\mathbf{5}$ with $S=8 / 2$, and 15.88 for $\mathbf{6}$ with $S=7 / 2$ (highestspin configurations) and as 4.99 for $\mathbf{1}$ with $S=\frac{1}{2}, 4.01$ for $\mathbf{2}$ with $S=\frac{1}{2}, 3.02$ for $\mathbf{3}$ with $S=\frac{0}{2}, 4.77$ and 4.78 for $\mathbf{4 a}$ and 4b with $S=1 / 2,4.99$ for $\mathbf{5}$ with $S=\frac{2}{2}$, and 3.76 for $\mathbf{6}$ with $S$ $=1 / 2$ (lowest-spin configurations). It is suggested from these results of $\left\langle\hat{S}^{2}\right\rangle \approx S(S+1)$ for the highest-spin configurations and $\left\langle\hat{S}^{2}\right\rangle \approx S(S+1)+5(\mathbf{1}), 4(\mathbf{2}, \mathbf{4 a}$, and $\mathbf{4 b})$, and $3(\mathbf{3}, \mathbf{5}$, and 6) for the lowest-spin configurations that the highest-spin


Top view


Side view
(a)


Top view



Side view
(b)


Top view

Side view
(c)

Figure 4. Optimized structures of iso-valence complexes in the lowest-spin configuration: (a) $\left.\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)\right)_{8}(\mathbf{1})$, (b) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}(\mathbf{2})$, and (c) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{4+}$ (3).


Top view

(a)


Top view

(b)


Top view

(c)


Top view

(d)

Figure 5. Optimized structures of mixed-valence complexes in the lowest-spin configuration: (a) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}(\mathbf{4 a})$, (b) $\left[\mathbf{M n} \mathrm{n}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}(\mathbf{4 b})$, (c) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}(5)$, and (d) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}(6)$.
configurations are nearly pure spin states and the lowest-spin configurations are the broken-symmetry states, which respectively include the five (1), four ( $\mathbf{2}, \mathbf{4} \mathbf{a}$, and $\mathbf{4 b})$, and three $(\mathbf{3}, \mathbf{5}$, and 6) pairs of singlet-coupled biradical electrons.

The relative energies indicate that the lowest-spin couplings are energetically more stable than the highest-spin couplings. The energy differences between both spin configurations for the complexes with the $\mathrm{Mn}(\mathrm{II})$ oxidation state are remarkably smaller than those for the complexes without the $\mathrm{Mn}(\mathrm{II})$ oxidation state: $0.2 \mathrm{kcal} / \mathrm{mol}$ for $\mathbf{1}$ with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II}), 0.6$ $\mathrm{kcal} / \mathrm{mol}$ for $\mathbf{4 a}$ and $\mathbf{4 b}$ with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$, and $0.1 \mathrm{kcal} / \mathrm{mol}$ for 5 with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})<7.1 \mathrm{kcal} / \mathrm{mol}$ for 2 with $\mathrm{Mn}($ III $)-$ Mn (III), $2.8 \mathrm{kcal} / \mathrm{mol}$ for $\mathbf{3}$ with $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$, and $5.7 \mathrm{kcal} /$ mol for 6 with $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$, suggesting that the magnetic interactions are weak in the Mn (II)-contained complexes.

## V. Stable Geometry of Model Complex

Figures 4 and 5, respectively, show the optimized structures of iso- and mixed-valence complexes in the lowest-spin configurations as Figures 4 a for 1, 4 b for 2, and 4 c for $\mathbf{3}$ and as Figures 5 a for $\mathbf{4 a}, 5$ b for $\mathbf{4 b}, 5$ c for $\mathbf{5}$, and 5 d for $\mathbf{6}$. We note that the optimized structures of complexes in the highest-spin configurations are not given in these figures because they have locations and orientations similar to those of $\mathrm{H}_{2} \mathrm{O}$ molecules at the lowest-spin configurations. The coordinated $\mathrm{H}_{2} \mathrm{O}$ ligands occupy the different positions in $\mathbf{1 , 4}$, and the others, respectively, and the hydrogen bonds are found in the optimized geometries except in $\mathbf{3}$. In this section, the numbering of the H and O atoms from 5-12 corresponds to the numbering of the $\mathrm{H}_{2} \mathrm{O}$ molecules indicated in Figure 3.

The $\mathrm{H}_{2} \mathrm{O}$ ligands of $\mathbf{1}$ move from the octahedral positions by rotating around the $\mathrm{Mn}-\mathrm{Mn}$ axis, and one of the H atoms in each of the $\mathrm{H}_{2} \mathrm{O}$ molecules faces the $\mu$-oxo ions to form hydrogen bonds with the $\mathrm{O}-\mathrm{H}$ distances of $R(\mathrm{O} 3-\mathrm{H} 5), R(\mathrm{O} 4-$ $\mathrm{H} 6), R(\mathrm{O} 4-\mathrm{H} 7)$, and $R(\mathrm{O} 3-\mathrm{H} 8)=1.694-1.697 \AA$ and $R(\mathrm{O} 3-$ $\mathrm{H} 9), R(\mathrm{O} 4-\mathrm{H} 10), R(\mathrm{O} 3-\mathrm{H} 11)$, and $R(\mathrm{O} 4-\mathrm{H} 12)=1.665-$ $1.668 \AA$ (lowest-spin configuration). Although the $\mathrm{H}_{2} \mathrm{O}$ ligands of 2 and 3 occupy the octahedral positions, their orientations differ from each other. The axial $\mathrm{H}_{2} \mathrm{O}$ ligands of 2 make the hydrogen bondings above and below the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core with $\mathrm{O}-\mathrm{H}$ distances of $R(\mathrm{O} 5-\mathrm{H} 7)=R(\mathrm{O} 8-\mathrm{H} 6)=1.982 \AA$ (lowest-spin configuration), whereas the $\mathrm{H}_{2} \mathrm{O}$ ligands of $\mathbf{3}$ seem to have no hydrogen bondings between $\mu$-oxo ions and $\mathrm{H}_{2} \mathrm{O}$ molecules or between $\mathrm{H}_{2} \mathrm{O}$ molecules.

It is characteristic for $\mathbf{4 a}$ and $\mathbf{4 b}$ that two types of hydrogen bonds are observed although they have different positions of $\mathrm{H}_{2} \mathrm{O}$ ligands. Two axial $\mathrm{H}_{2} \mathrm{O}$ molecules are located at the octahedral positions of Mn1 (4a) or Mn2 (4b) corresponding to $\mathrm{Mn}(\mathrm{II})$ or $\mathrm{Mn}(\mathrm{III})$, whereas the other axial $\mathrm{H}_{2} \mathrm{O}$ molecules seem to coordinate to $\mu$-oxo ions rather than to Mn ions. The hydrogen bonds are formed between axial $\mathrm{H}_{2} \mathrm{O}$ ligands above and below the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core, and the axial $\mathrm{H}_{2} \mathrm{O}$ ligands shifted from the octahedral positions also make hydrogen bonds with $\mu$-oxo ions. The hydrogen bond $\mathrm{O}-\mathrm{H}$ distances are obtained as follows: $R(\mathrm{O} 3-\mathrm{H} 7)=1.709 \AA$ and $R(\mathrm{O} 4-\mathrm{H} 8)=1.700 \AA$ between the $\mu$-oxo ion and the $\mathrm{H}_{2} \mathrm{O}$ molecule and $R(\mathrm{O} 7-\mathrm{H} 5)$ $=1.763 \AA$ and $R(\mathrm{O} 8-\mathrm{H} 6)=1.731 \AA$ between $\mathrm{H}_{2} \mathrm{O}$ molecules (lowest-spin configuration of $\mathbf{4 a}$ ); $R(\mathrm{O} 3-\mathrm{H} 5)=1.643 \AA$ and $R(\mathrm{O} 4-\mathrm{H} 6)=1.642 \AA$ between the $\mu$-oxo ion and $\mathrm{H}_{2} \mathrm{O}$ molecule and $R(\mathrm{O} 5-\mathrm{H} 7)=1.972 \AA$ and $R(\mathrm{O} 6-\mathrm{H} 8)=1.971 \AA$ between $\mathrm{H}_{2} \mathrm{O}$ molecules (lowest-spin configuration of $\mathbf{4 b}$ ). We note that although several energy minima of 4 , in which the $\mathrm{H}_{2} \mathrm{O}$ ligands are fully placed at the octahedral positions and the hydrogen bonds are formed between axial $\mathrm{H}_{2} \mathrm{O}$ molecules similar to 2 , were found, their structures were higher in energy than $\mathbf{4 a}$ and 4b by more than $\sim 7 \mathrm{kcal} / \mathrm{mol}$ as described in section IV. The octahedral geometries of $\mathrm{H}_{2} \mathrm{O}$ ligands are favorable to 5 and $\mathbf{6}$, and hydrogen bonds are found between axial $\mathrm{H}_{2} \mathrm{O}$ molecules as seen in 2. The hydrogen bond $\mathrm{O}-\mathrm{H}$ distances are obtained as follows: $R(\mathrm{O} 5-\mathrm{H} 7)=R(\mathrm{O} 6-\mathrm{H} 8)=1.924 \AA$ (lowest-spin configuration of 5); $R(\mathrm{O} 5-\mathrm{H} 7)=1.857 \AA$ and $R(\mathrm{O} 6-\mathrm{H} 8)=$ $1.855 \AA$ (lowest-spin configuration of $\mathbf{6}$ ).

Although the axial $\mathrm{H}_{2} \mathrm{O}$ ligands of $\mathbf{2}$ and $\mathbf{4 - 6}$ form hydrogen bonds with each other as mentioned above, the roles are different between 2 and 4-6 in the most stable coordinations. Two of the axial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to the same Mn ion play different and the same roles as the hydrogen donor or hydrogen acceptor in $\mathbf{2}$ and $\mathbf{4 - 6}$, respectively. The axial $\mathrm{H}_{2} \mathrm{O}$ ligands of $\mathbf{2 , 5}$, and $\mathbf{6}$ at the octahedral positions have shorter $\mathrm{Mn}-\mathrm{OH}_{2}$ distances for the hydrogen donor than they do for the hydrogen accepter as $\mathrm{Mn}-\mathrm{OH}_{2}$ (donor) $\approx 2.25 \AA<\mathrm{Mn}-\mathrm{OH}_{2}$ (acceptor) $\approx 2.41 \AA$ for $2, \mathrm{Mn}-\mathrm{OH}_{2}$ (donor) $\approx 2.01 \AA<\mathrm{Mn}-\mathrm{OH}_{2}$ $($ acceptor $) \approx 2.33 \AA$ for 5 , and $\mathrm{Mn}-\mathrm{OH}_{2}$ (donor) $\approx 1.97 \AA<$ $\mathrm{Mn}-\mathrm{OH}_{2}$ (acceptor) $\approx 2.36 \AA$ for 6 (lowest-spin configuration), where the $\mathrm{H}_{2} \mathrm{O}$ molecules of the hydrogen donor (acceptor) are coordinated to the higher (lower) oxidation site of the $\mathrm{Mn}-\mathrm{Mn}$ center in the mixed-valence complexes 5 and 6.

We compared the hydrogen bonds found in $\mathbf{1 , 2}$, and $\mathbf{4 - 6}$. The hydrogen bond lengths between the $\mu$-oxo ion and the $\mathrm{H}_{2} \mathrm{O}$ molecule are about 1.70 and $1.67 \AA$ in $\mathbf{1}$, about $1.70 \AA$ in $\mathbf{4 a}$, and about $1.64 \AA$ in $\mathbf{4 b}$, which are shorter, with the exception of $\mathbf{4 a}$, than the hydrogen-bond lengths between the $\mathrm{H}_{2} \mathrm{O}$ molecules ranging from about $1.86-1.98 \AA$ in $\mathbf{2 , 4 b}, \mathbf{5}$, and $\mathbf{6}$. These additional hydrogen bonds would be the reason why $\mathbf{4 a}$
and $\mathbf{4 b}$ have lower energies than the other energy minima of $\mathbf{4}$. The hydrogen bonds between $\mathrm{H}_{2} \mathrm{O}$ molecules are shorter in $\mathbf{4 a}$ $(\sim 1.73$ and $1.76 \AA)$ than they are in $2(\sim 1.98 \AA), \mathbf{4 b}(\sim 1.97$ $\AA), 5(\sim 1.92 \AA)$, and $\mathbf{6}(\sim 1.86 \AA)$.

With the exception of the complex including the $\mathrm{Mn}(\mathrm{II})-$ Mn (IV) core (5), the most-stable coordination positions of the $\mathrm{H}_{2} \mathrm{O}$ molecules are different between the complexes. The Mn(II) ion (1 and 4) has some shifted $\mathrm{H}_{2} \mathrm{O}$ ligands, and the complexes without the $\mathrm{Mn}(\mathrm{II})$ ion (2, 3, and 6) have eight octahedral $\mathrm{H}_{2} \mathrm{O}$ ligands. Because all of the octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals, which indicate the antibonding nature between Mn ions and both $\mathrm{H}_{2} \mathrm{O}$ ligands in the axial and equatorial directions, are singly occupied in the Mn (II) oxidation state, it is expected that the $\mathrm{Mn}(\mathrm{II})-\mathrm{OH}_{2}$ bindings become loose. It is thus likely that the $\mathrm{H}_{2} \mathrm{O}$ ligands coordinated to the $\mathrm{Mn}(\mathrm{II})$ ion would move from the octahedral positions to the more stable positions, which enable them to make hydrogen bonds with $\mu$-oxo ions.

Figure 6 shows the optimized interatomic distances and angles of iso- and mixed-valence complexes in the lowest-spin configuration as Figures 6a for $\mathrm{Mn}-\mathrm{Mn}, 6 \mathrm{~b}$ for $\mathrm{O}-\mathrm{O}, 6 \mathrm{c}$ for $\mathrm{Mn}-$ $\mathrm{O}, 6 \mathrm{~d}$ for $\mathrm{Mn}-\mathrm{OH}_{2}$ (axial), 6 e for $\mathrm{Mn}-\mathrm{OH}_{2}$ (equatorial), 6 f for $\mathrm{Mn}-\mathrm{O}-\mathrm{Mn}$, and 6 g for $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$. The optimized values of these parameters for the complexes in both of the highestand lowest-spin configurations are given in Appendix A. In Figures $6 \mathrm{a}-6 \mathrm{~g}$, the oxidation states of the $\mathrm{Mn} 1-\mathrm{Mn} 2$ center in the mixed-valence complexes correspond to Mn (II) -Mn (III) for $\mathbf{4 a}$ and $\mathbf{4 b}, \mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ for $\mathbf{5}$, and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ for 6, respectively.

It is found from Figure 6a that the $\mathrm{Mn}-\mathrm{Mn}$ separations seem to have no relation to the formal oxidation states of Mn ions as Mn (II) -Mn (II) in $\mathbf{1}>\mathrm{Mn}$ (II) -Mn (III) in $\mathbf{4 a}$ and $\mathbf{4 b}>\mathrm{Mn}-$ (IV) $-\mathrm{Mn}(\mathrm{IV})$ in $\mathbf{3}$ and $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ in $5>\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}-$ (IV) in $6>\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}($ III $)$ in 2, although the order of the $\mathrm{Mn}-\mathrm{Mn}$ lengths may be expected simply from the electrostatic repulsion between the formal charges on Mn ions as $\mathrm{Mn}(\mathrm{II})-$ $\mathrm{Mn}(\mathrm{II})$ in $\mathbf{1}<\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (III) in $\mathbf{4 a}$ and $\mathbf{4 b}<\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}-$ (IV) in $\mathbf{5}<\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}($ III $)$ in $\mathbf{2}<\mathrm{Mn}($ III $)-\mathrm{Mn}(\mathrm{IV})$ in $6<$ $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ in 3. The $\mathrm{O}-\mathrm{O}$ and $\mathrm{Mn}-\mathrm{O}$ distances shown in Figures 6 b and 6 c also vary depending on the oxidation states of complexes as $\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{II})_{2} \mathrm{O}_{2}$ core in $\mathbf{1}>\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{II}) \mathrm{Mn}(\mathrm{III}) \mathrm{O}_{2}$ core in $\mathbf{4 a}$ and $\mathbf{4 b}>\mathrm{O}-\mathrm{O}$ for the Mn (II) $\mathrm{Mn}($ IV $) \mathrm{O}_{2}$ core in $5>\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{III})_{2} \mathrm{O}_{2}$ core in $2>$ $\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{III}) \mathrm{Mn}(\mathrm{IV}) \mathrm{O}_{2}$ core in $\mathbf{6}>\mathrm{O}-\mathrm{O}$ for the Mn $(\mathrm{IV})_{2} \mathrm{O}_{2}$ core in $\mathbf{3}$ and as $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ in $\mathbf{1}, \mathbf{4 a}, \mathbf{4 b}$, and $\mathbf{5}>\mathrm{Mn}-$ (III) -O in 2, 4a, 4b, and $\mathbf{6}$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ in 3, 5, and 6. The $\mathrm{Mn}(\mathrm{II})$ centers have the $\mathrm{Mn}-\mathrm{O}$ lengths near $2.1 \AA$, whereas the $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$ centers have the $\mathrm{Mn}-\mathrm{O}$ lengths around 1.7-1.8 A, suggesting that more repulsive interactions would act between Mn (II) and $\mu$-oxo ions than they would between Mn (III) or Mn (IV) and $\mu$-oxo ions. The coordination distances of the octahedral $\mathrm{H}_{2} \mathrm{O}$ ligands to the Mn ions given in Figures 6 d and 6 e indicate the features that $\mathrm{Mn}(\mathrm{II})-\mathrm{OH}_{2}$ in $\mathbf{4 a}$ and 5 and $\mathrm{Mn}(\mathrm{III})-\mathrm{OH}_{2}$ in $\mathbf{2}, \mathbf{4 b}$, and $\mathbf{6}>\mathrm{Mn}(\mathrm{IV})-\mathrm{OH}_{2}$ in $\mathbf{3}, \mathbf{5}$, and 6 for the axial positions (Figure 6 d ) and $\mathrm{Mn}(\mathrm{II})-\mathrm{OH}_{2}$ in $\mathbf{4 a}$, 4b, and $5>\mathrm{Mn}(\mathrm{III})-\mathrm{OH}_{2}$ in 2, 4a, 4b, and $\mathbf{6}$ and $\mathrm{Mn}(\mathrm{IV})-$ $\mathrm{OH}_{2}$ in 3,5, and $\mathbf{6}$ for the equatorial positions (Figure 6e), although the differences are smaller for the equatorial ligands, especially in $\mathbf{4 b}$ and $\mathbf{5}$, than they are for the axial ligands.

Figure 6 f indicates that the higher the Mn ions are oxidized, the larger the $\mathrm{Mn}-\mathrm{O}-\mathrm{Mn}$ angles become for the iso-valence complexes, and the $\mathrm{Mn}-\mathrm{O}-\mathrm{Mn}$ angles for the mixed-valence complexes lie between the values in the corresponding isovalence states, which include one of the component Mn ions in the mixed-valence states as follows: $\mathrm{Mn}(\mathrm{II})-\mathrm{O}-\mathrm{Mn}(\mathrm{II})$ in 1


Figure 6. Optimized interatomic distances and angles of iso- and mixed-valence complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the lowest-spin configuration: (a) $\mathrm{Mn}-\mathrm{Mn}$, (b) $\mathrm{O}-\mathrm{O}$, (c) $\mathrm{Mn}-\mathrm{O}$, (d) $\mathrm{Mn}-\mathrm{OH}_{2}$ (axial), (e) $\mathrm{Mn}-\mathrm{OH}_{2}$ (equatorial), (f) $\mathrm{Mn}-\mathrm{O}-\mathrm{Mn}$, and (g) $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$.

TABLE 2: Experimental and Theoretical Structures of the $\mathrm{Mn}_{2} \mathrm{O}_{\mathbf{2}}$ Core in Iso-Valence $\mathbf{M n}_{2} \mathrm{O}_{\mathbf{2}}$ Complexes

| oxidation state | $\mathrm{Mn}-\mathrm{Mn}^{a}$ | $\mathrm{O}-\mathrm{O}^{a}$ | $\mathrm{Mn}-\mathrm{O}^{a}$ | $\mathrm{O}-\mathrm{Mn}-\mathrm{O}^{b}$ | ref |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | experiment |  |  |  |
| $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ | 2.674-2.686 |  | 1.830-1.842 | 86.1-86.7 | 30 |
|  | 2.674-2.699 | 2.500-2.526 | 1.830-1.848 |  | 34 |
| $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ | 2.672-2.748 |  | $1.774-1.811$ | $78.5-85.0$ | 30 |
|  | $2.568-2.780$ | 2.246-2.538 | $1.774-1.834$ |  | 34 |
| calculation |  |  |  |  |  |
| $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (II) | 2.890 | 2.978 | 2.073-2.076 | 91.7 | this work |
| $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ | 2.651 | 2.397 | 1.784, 1.790 | 84.2 | this work |
|  | 2.716 |  | $1.840$ | 84.8 | $30$ |
|  | 2.791 | 2.433 | 1.852 |  | 34 |
|  | 2.690 | 2.465 | 1.797, 1.851 |  | 37 |
| $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ | 2.762 | 2.249 | 1.781 | $78.3$ | this work |
|  | 2.866 |  | 1.842 | 77.9 | $30$ |
|  | $2.902$ | 2.283 | 1.846 |  | 34 |
|  | 2.834 | 2.272 | 1.789, 1.842 |  | 37 |

${ }^{a}$ Interatomic distances are given in angströms. ${ }^{b}$ Interatomic angles are given in degrees.
$<\mathrm{Mn}(\mathrm{II})-\mathrm{O}-\mathrm{Mn}(\mathrm{III})$ in $\mathbf{4 a}$ and $\mathbf{4 b}$ and $\mathrm{Mn}(\mathrm{II})-\mathrm{O}-\mathrm{Mn}(\mathrm{IV})$ in $\mathbf{5}<\mathrm{Mn}$ (III) $-\mathrm{O}-\mathrm{Mn}$ (III) in $\mathbf{2}<\mathrm{Mn}$ (III) $-\mathrm{O}-\mathrm{Mn}$ (IV) in $\mathbf{6}$ $<\mathrm{Mn}(\mathrm{IV})-\mathrm{O}-\mathrm{Mn}(\mathrm{IV})$ in 3. It is found from Figure 6 g that the $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$ angle has the tendency to become more acute for the higher-oxidized Mn ion compared among the iso-valence complexes and has the tendency to become more obtuse for the higher-oxidized Mn ion compared within the mixed-valence complex as follows: $\mathrm{O}-\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ in $\mathbf{1}>\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}$ in 2 $>\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ in $\mathbf{3}$ (iso-valence complexes); $\mathrm{O}-\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ $<\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}$ in $\mathbf{4 a}$ and $\mathbf{4 b}, \mathrm{O}-\mathrm{Mn}(\mathrm{II})-\mathrm{O}<\mathrm{O}-\mathrm{Mn}-$ (IV) -O in 5 , and $\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}<\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ in 6 (mixedvalence complexes).

These changes in molecular geometry, depending on the oxidation state of complexes described above, would be caused
by the changes in electron occupation of the $\mathrm{Mn}-\mathrm{O}$ and $\mathrm{Mn}-$ $\mathrm{OH}_{2}$ antibonding orbitals with the oxidation of the $\mathrm{Mn}-\mathrm{Mn}$ center.

Some of the geometrical parameters of di- $\mu$-oxo-bridged $\mathrm{Mn}_{2} \mathrm{O}_{2}$ complexes with various ligands (mainly N -donor) observed by experiments and those of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{6}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}\right]^{q+}$ calculated by the broken-symmetry DFT method within the spatial symmetry framework have been summarized in refs 30,34 , and 37 , and the available data for the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core in the iso- and mixed-valence oxidation states are listed in Tables 2 and 3, respectively, together with the results of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ determined in this work. In these Tables, the ranges of the experimental values and the theoretical

TABLE 3: Experimental and Theoretical Structures of the $\mathbf{M n}_{2} \mathbf{O}_{2}$ Core in Mixed-Valence $\mathbf{M n}_{2} \mathbf{O}_{\mathbf{2}}$ Complexes

| oxidation state | $\mathrm{Mn}-\mathrm{Mn}^{\text {a }}$ | $\mathrm{O}-\mathrm{O}^{a}$ | $\mathrm{Mn}-\mathrm{O}^{a, c}$ | $\mathrm{Mn}-\mathrm{O}^{\text {a,d }}$ | $\mathrm{O}-\mathrm{Mn}-\mathrm{O}^{\text {b,c }}$ | $\mathrm{O}-\mathrm{Mn}-\mathrm{O}^{\text {b,d }}$ | ref |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| experiment |  |  |  |  |  |  |  |
| $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ | 2.643-2.741 |  | 1.833-1.862 | 1.774-1.793 | 80.6-82.5 | 84.6-86.8 | 30 |
|  | 2.553-2.741 | $2.373-2.525$ | 1.81-1.89 | $1.77-1.81$ |  |  | 34 |
| calculation |  |  |  |  |  |  |  |
| $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$ | 2.828 | 2.606 | 2.103, 2.106 | $1.754,1.755$ | 76.5 | 95.9 | this work this work 34 |
|  | 2.814 | 2.628 | 2.063, 2.065 | 1.795 | 79.0 | 94.0 |  |
|  | 2.833 | 2.635 | 2.063 | 1.813 |  |  |  |
| $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ | 2.749 | 2.517 | 2.048 | 1.694 | 75.8 | 96.0 | this work |
| Mn (III)-Mn(IV) | 2.703 | 2.318 | 1.822, 1.827 | 1.733, 1.740 | 78.9 | 83.7 | this work |
|  | 2.783 |  | 1.873 | 1.800 | 79.5 | 85.2 | 30 |
|  | 2.845 | 2.358 | 1.921 | 1.777 |  |  | 34 |
|  | 2.761 | 2.353 | 1.773-1.864 |  |  |  | 37 |

${ }^{a}$ Interatomic distances are given in angströms. ${ }^{b}$ Interatomic angles are given in degrees. ${ }^{c}$ Geometrical parameters for the lower oxidation state of Mn ion are shown. ${ }^{d}$ Geometrical parameters for the higher oxidation state of Mn ion are shown.
values optimized for the most-stable broken-symmetry states are given concerning the $\mathrm{Mn}-\mathrm{Mn}, \mathrm{O}-\mathrm{O}, \mathrm{Mn}-\mathrm{O}$, and $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$ parameters.

Our results are comparable with the reported experimental and theoretical results, and the changes in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ structure as $\mathrm{Mn}($ III $)-\mathrm{Mn}($ III $)<\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})<\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$, $\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{III})_{2} \mathrm{O}_{2}$ core $>\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{III}) \mathrm{Mn}(\mathrm{IV})-$ $\mathrm{O}_{2}$ core $>\mathrm{O}-\mathrm{O}$ for the $\mathrm{Mn}(\mathrm{IV})_{2} \mathrm{O}_{2}$ core, $\mathrm{Mn}(\mathrm{III})-\mathrm{O} \approx \mathrm{Mn}-$ (IV) -O and $\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}>\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ for the $\mathrm{Mn}-$ $(\mathrm{III})_{2} \mathrm{O}_{2}$ and $\mathrm{Mn}(\mathrm{IV})_{2} \mathrm{O}_{2}$ cores, and $\mathrm{Mn}(\mathrm{III})-\mathrm{O}>\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ and $\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}<\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ for the $\mathrm{Mn}($ III $) \mathrm{Mn}(\mathrm{IV})-$ $\mathrm{O}_{2}$ core are consistent among the calculations in this work and in the other work. Although the experimental data are lacking for the complexes containing $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$, and $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ centers, the theoretical geometries of Mn (II) -Mn (III) complex in the present study correspond to those in the early study.

From comparison of the optimized geometries in the Mn(III) -Mn (III), $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$, and Mn (III) -Mn (IV) oxidation states between $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in this work and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{9+}$ shown in ref 30 , it is found that the $\mathrm{Mn}-\mathrm{O}$ distances in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core become shorter in complexes with $\mathrm{H}_{2} \mathrm{O}$ ligands than in complexes with $\mathrm{NH}_{3}$ ligands by about $0.05-$ $0.07 \AA$, whereas the $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$ angles are almost the same in both types of complex, and in consequence the $\mathrm{Mn}-\mathrm{Mn}$ separations are shorter in the former than they are in the latter. The distances between Mn ions and $\mathrm{NH}_{3}$ ligands in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ were reported in ref 30 as follows for the axial and equatorial positions: 2.504 and $2.231 \AA$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}-$ (III) complex, 2.137 and $2.215 \AA$ in the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ complex, 2.459 and $2.210 \AA$ for $\mathrm{Mn}(\mathrm{IIII})$; and 2.117 and 2.227 $\AA$ for $\mathrm{Mn}(\mathrm{IV})$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ complex. It was also pointed out in ref 30 that the axial $\mathrm{Mn}-\mathrm{NH}_{3}$ length at the Mn (III) site in the mixed-valence system ( $2.459 \AA$ ) is significantly shorter than that in the iso-valence system ( $2.504 \AA$ ), whereas the structural characteristics of the $\mathrm{Mn}(\mathrm{IV})$ site in the mixedvalence system are almost identical to those in the iso-valence system, suggesting that an extra electron in the mixed-valence complex is not completely localized in the $\mathrm{d}_{z^{2}}$ orbital on the Mn (III) center and, however, its electron density is not transferred into the $\mathrm{d}_{z}{ }^{2}$ orbital on the $\mathrm{Mn}(\mathrm{IV})$ center (The $C_{2 v}$ restriction was applied to the spatial symmetry of model complexes in ref 30). In the case of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$, the $\mathrm{H}_{2} \mathrm{O}$ ligands have the following distances from Mn ions: 2.250 and $2.251 \AA$ (axial hydrogen donor), 2.405 and $2.406 \AA$ (axial hydrogen acceptor), and 2.098 and $2.100 \AA$ (equatorial) in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ complex; $1.968 \AA$ (axial) and $2.049 \AA$ (equatorial) in the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ complex; 2.356 and 2.357


Figure 7. X-ray structure of the cubanelike $\mathrm{Mn}_{3} \mathrm{CaO}_{4}-\mathrm{Mn}$ cluster in the OEC (pdb code: 1S5L) ${ }^{19}$ and numbering of atoms.

TABLE 4: X-ray Interatomic Distances for $\mathbf{M n}-\mathbf{M n}, \mathbf{O}-\mathbf{O}$, and $\mathbf{M n}-\mathrm{O}$ Pairs of the Cubanelike $\mathrm{Mn}_{3} \mathrm{CaO}_{4}-\mathrm{Mn}$ Cluster in the OEC (pdb code: 1S5L) ${ }^{19}$

| $\mathrm{Mn}-\mathrm{Mn}($ angströms) | $\mathrm{O}-\mathrm{O}$ (angströms) |  | $\mathrm{Mn}-\mathrm{O}$ (angströms) |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Mn}(1)-\mathrm{Mn}(2)$ | 2.648 | $\mathrm{O}(1)-\mathrm{O}(2)$ | 2.500 | $\mathrm{Mn}(1)-\mathrm{O}(1)$ | 1.872 |
| $\mathrm{Mn}(1)-\mathrm{Mn}(3)$ | 2.669 | $\mathrm{O}(1)-\mathrm{O}(3)$ | 2.694 | $\mathrm{Mn}(1)-\mathrm{O}(2)$ | 1.824 |
| $\mathrm{Mn}(1)-\mathrm{Mn}(4)$ | 5.127 | $\mathrm{O}(1)-\mathrm{O}(4)$ | 2.661 | $\mathrm{Mn}(1)-\mathrm{O}(3)$ | 1.863 |
| $\mathrm{Mn}(2)-\mathrm{Mn}(3)$ | 2.718 | $\mathrm{O}(2)-\mathrm{O}(3)$ | 2.487 | $\mathrm{Mn}(2)-\mathrm{O}(1)$ | 1.770 |
| $\mathrm{Mn}(2)-\mathrm{Mn}(4)$ | 3.249 | $\mathrm{O}(2)-\mathrm{O}(4)$ | 2.563 | $\mathrm{Mn}(2)-\mathrm{O}(2)$ | 1.821 |
| $\mathrm{Mn}(3)-\mathrm{Mn}(4)$ | 3.255 | $\mathrm{O}(3)-\mathrm{O}(4)$ | 2.696 | $\mathrm{Mn}(2)-\mathrm{O}(4)$ | 1.938 |
|  |  |  |  | $\mathrm{Mn}(3)-\mathrm{O}(2)$ | 1.802 |
|  |  |  |  | $\mathrm{Mn}(3)-\mathrm{O}(3)$ | 1.809 |
|  |  |  | $\mathrm{Mn}(3)-\mathrm{O}(4)$ | 1.912 |  |
|  |  |  | $\mathrm{Mn}(4)-\mathrm{O}(4)$ | 1.866 |  |

$\AA$ (axial hydrogen acceptor) and 2.051 and $2.058 \AA$ (equatorial) for $\mathrm{Mn}(\mathrm{III})$ and 1.969 (axial hydrogen donor) and 2.066 and $2.072 \AA$ (equatorial) for $\mathrm{Mn}(\mathrm{IV})$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ complex, as shown in Appendix A (Table A3). Although the distances between Mn ions and axial or equatorial $\mathrm{H}_{2} \mathrm{O}$ ligands in these oxidation states also become shorter in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ than in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ by about $0.1-0.15 \AA$, they indicate similar geometrical characteristics to those of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ described above for the axial $\mathrm{H}_{2} \mathrm{O}$ ligands of the hydrogen acceptor at the $\mathrm{Mn}(\mathrm{III})$ site and the $\mathrm{H}_{2} \mathrm{O}$ ligands at the $\mathrm{Mn}(\mathrm{IV})$ site.

Here, we compare the theoretical and experimental $\mathrm{Mn}-\mathrm{Mn}$, $\mathrm{O}-\mathrm{O}$, and $\mathrm{Mn}-\mathrm{O}$ distances among our model complexes and the cubanelike cluster in the OEC. The X-ray structure ${ }^{19}$ and X-ray distances are shown in Figure 7 and in Table 4, respectively. It is noted that because the resolution of the X-ray crystal structure is $3.5 \AA$ and the model complexes are Mn -

TABLE 5: Mulliken Charge and Spin Densities on Atoms in the $\mathbf{M n}_{2} \mathbf{O}_{2}$ Core of Iso- and Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{\mathbf{2}}\left(\mathbf{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$

| complex | $S$ | charge density |  |  |  | spin density |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mn1 | Mn2 | O3 | O4 | Mn1 | Mn2 | O3 | O4 |
| 1 | 10/2 | 1.297 | 1.297 | -1.109 | -1.108 | 4.756 | 4.756 | 0.138 | 0.137 |
|  | 0/2 | 1.294 | 1.294 | -1.111 | -1.112 | 4.754 | -4.754 | -0.000 | 0.001 |
| 2 | 8/2 | 1.537 | 1.536 | -0.908 | -0.908 | 3.866 | 3.865 | 0.070 | 0.070 |
|  | $0 / 2$ | 1.521 | 1.521 | -0.889 | -0.889 | 3.814 | -3.814 | 0.003 | -0.003 |
| 3 | 6/2 | 1.725 | 1.725 | -0.665 | -0.665 | 3.024 | 3.024 | -0.046 | -0.046 |
|  | 0/2 | 1.718 | 1.718 | -0.660 | -0.660 | 2.871 | -2.871 | -0.000 | 0.000 |
| 4a | $9 / 2$ | 1.372 | 1.410 | -1.008 | -1.009 | 4.791 | 3.785 | 0.147 | 0.147 |
|  | 1/2 | 1.375 | 1.408 | $-1.010$ | -1.011 | 4.762 | -3.800 | -0.018 | -0.018 |
| 4b | 9/2 | 1.366 | 1.480 | $-1.043$ | -1.043 | 4.773 | 3.822 | 0.117 | 0.118 |
|  | 1/2 | 1.364 | 1.478 | $-1.044$ | -1.043 | 4.746 | -3.849 | 0.044 | 0.044 |
| 5 | 8/2 | 1.408 | 1.566 | -0.891 | -0.891 | 4.772 | 2.662 | 0.250 | 0.250 |
|  | 2/2 | 1.414 | 1.565 | -0.895 | -0.895 | 4.740 | -2.615 | -0.114 | -0.114 |
| 6 | 7/2 | 1.604 | 1.683 | -0.759 | -0.789 | 3.906 | 2.938 | 0.050 | 0.044 |
|  | 1/2 | 1.599 | 1.664 | -0.742 | -0.772 | 3.939 | -2.676 | -0.201 | $-0.171$ |

Mn binuclear complexes the comparisons of theoretical and experimental results are therefore qualitative.

The cubanelike faces of the Mn cluster in the OEC are composed of $\mathrm{Mn}-\mathrm{Mn}$ pairs of $\mathrm{Mn}(1)-\mathrm{Mn}(2), \mathrm{Mn}(1)-\mathrm{Mn}(3)$, and $\mathrm{Mn}(2)-\mathrm{Mn}(3)$ as seen in Figure 7, and these pairs have distances of $2.648,2.669$, and $2.718 \AA$ for the $\mathrm{Mn}-\mathrm{Mn}$ separation, respectively. The optimized separations for $\mathrm{Mn}($ III $)-$ Mn (III), $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}$ (IV), $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (IV), and Mn (III) $-\mathrm{Mn}-$ (IV) in $\mathbf{2}, \mathbf{3}, \mathbf{5}$, and $\mathbf{6}$ are respectively comparable with the experimental values as $2.651,2.762,2.749$, and $2.703 \AA$, whereas the theoretical Mn (II) -Mn (II) and Mn (II) -Mn (III) distances of $2.890,2.828$, and $2.814 \AA$ in $\mathbf{1}, \mathbf{4 a}$, and $\mathbf{4 b}$ are much longer than the experimental distances.

Concerning the $\mathrm{O}-\mathrm{O}$ distances of di- $\mu$-oxo bridges within cubanelike faces, the experimental values were reported as $\mathrm{O}(1)-\mathrm{O}(2)=2.500 \AA, \mathrm{O}(2)-\mathrm{O}(3)=2.487 \AA$, and $\mathrm{O}(2)-\mathrm{O}(4)$ $=2.563 \AA$. However, the $\mathrm{O}(1), \mathrm{O}(3)$, and $\mathrm{O}(4)$ ions bond with the Ca ion, and the $\mathrm{O}(4)$ ion bonds with the outer $\mathrm{Mn}(4)$ ion. It is thus considered that the experimental $\mathrm{O}-\mathrm{O}$ lengths are strongly affected by these ions. Although the theoretical values for iso-valence complexes $\mathbf{2}$ and $\mathbf{3}$ are shorter than the experimental values like $\mathrm{O}-\mathrm{O}$ in 2 with the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ center $=2.397 \AA$ and $\mathrm{O}-\mathrm{O}$ in $\mathbf{3}$ with the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ center $=2.249 \AA$, among interatomic separations, the tendency for the $\mathrm{Mn}-\mathrm{Mn}$ bond to be longer than the $\mathrm{O}-\mathrm{O}$ bond is consistent between the experiments and calculations. The optimized value for iso-valence complex 1 with the $\mathrm{Mn}(\mathrm{II})-$ Mn (II) center indicates that the theoretical $\mathrm{O}-\mathrm{O}$ length of 2.978 $\AA$ is much longer than the experimental lengths and is also longer than the theoretical $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (II) length. The theoretical values of $\mathrm{O}-\mathrm{O}$ separations are $2.606,2.628,2.517$, and 2.318 $\AA$, respectively, for mixed-valence complexes $\mathbf{4 a}, \mathbf{4 b}, \mathbf{5}$, and $\mathbf{6}$. The distance for 5 in the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ oxidation state shows similarity to the experimental values, and the distances for $\mathbf{4 a}$ and 4 b in the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$ oxidation state and for $\mathbf{6}$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ oxidation state seem to be longer and shorter than the experimental values, respectively.

The experimental $\mathrm{Mn}-\mathrm{O}$ lengths range from 1.770 to 1.938 $\AA$. The $\mathrm{Mn}(2)-\mathrm{O}(4)$ and $\mathrm{Mn}(3)-\mathrm{O}(4)$ give longer separations of 1.938 and $1.912 \AA$, respectively, in which the $\mathrm{O}(4)$ ion interacts with the $\mathrm{Mn}(4)$ and Ca ions. The $\mathrm{Mn}(1)-\mathrm{O}(1)$ and $\mathrm{Mn}(1)-\mathrm{O}(3)$ also give longer separations of 1.872 and 1.863 $\AA$, respectively, in which the $\mathrm{O}(1)$ and $\mathrm{O}(3)$ ions interact with the Ca ion. The $\mathrm{Mn}(1)-\mathrm{O}(2), \mathrm{Mn}(2)-\mathrm{O}(1), \mathrm{Mn}(2)-\mathrm{O}(2), \mathrm{Mn}-$ (3) $-\mathrm{O}(2)$, and $\mathrm{Mn}(3)-\mathrm{O}(3)$ give shorter separations of 1.824 , $1.770,1.821,1.802$, and $1.809 \AA$, respectively. The theoretical $\mathrm{Mn}-\mathrm{O}$ distances obtained are 1.784 and $1.790 \AA$ for 2, 1.781
$\AA$ for $\mathbf{3}$, and $2.073-2.076 \AA$ for $\mathbf{1}$, indicating that the results for $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ complexes show similarity to the shorter experimental values. The optimized $\mathrm{Mn}-\mathrm{O}$ distances of $2.063-2.106 \AA$ for $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ and 1.754$1.795 \AA$ for $\mathrm{Mn}(\mathrm{III})-\mathrm{O}$ in $\mathbf{4 a}$ and $\mathbf{4 b}, 2.048 \AA$ for $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ and $1.694 \AA$ for $\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ in 5, and 1.822 and $1.827 \AA$ for $\mathrm{Mn}(\mathrm{III})-\mathrm{O}$ and 1.733 and $1.740 \AA$ for $\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ in 6 suggest that the $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ in $\mathbf{4 a}, \mathbf{4 b}$, and $\mathbf{5}$ and the $\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ in $\mathbf{5}$ would be too long and too short in comparison with the experimental distances, respectively, whereas the calculated Mn(III) -O and $\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ separations in 6 may be comparable to the experimental values of the $\mathrm{Mn}(1)-\mathrm{O}(1)-\mathrm{Mn}(2)-\mathrm{O}(2)$ or the $\mathrm{Mn}(1)-\mathrm{O}(2)-\mathrm{Mn}(3)-\mathrm{O}(3)$ cubanelike faces.

Judging from the comparison of interatomic distances in the experiments and calculations described above, it may be possible to suggest that the cubanelike $\mathrm{Mn}_{3} \mathrm{CaO}_{4}$ cluster in the OEC contains the low-spin-coupled $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}-$ (IV), or $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ pairs, and the Mn (II) oxidation state is not included.

## VI. Mulliken Population Analysis of Model Complex

We summarize the charge and spin densities on atoms in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core of the iso- and mixed-valence complexes in Table 5. Tables 6 and 7 also show the gross orbital populations in the Mn 3 d orbitals of the iso- and mixed-valence complexes in the lowest-spin configuration, respectively. As described in the next section, the octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals of the $\mathrm{Mn}-\mathrm{Mn}$ center in the axial direction are assigned for each complex to be $\mathrm{d}_{y^{2}-z^{2}}$ for $\mathbf{1}$, $\mathrm{d}_{x^{2}-z^{2}}$ for $\mathbf{2}, \mathrm{d}_{z^{2}}$ for $\mathbf{3}, \mathrm{d}_{x^{2}-z^{2}}$ on Mn (II) and $\mathrm{d}_{y^{2}-z^{2}}$ on Mn (III) for 4, $\mathrm{d}_{z^{2}}$ on Mn (II) and Mn (IV) for 5 , and $\mathrm{d}_{y^{2}}{ }^{2} z^{2}$ on Mn (III) and $\mathrm{d}_{z^{2}}$ on Mn (IV) for 6 .

The charge densities on the Mn and $\mu$-oxo ions in the lowestspin configuration are respectively obtained as follows: 1.29 for $\mathrm{Mn}(\mathrm{II})$ and -1.11 for O in $\mathbf{1} ; 1.52$ for $\mathrm{Mn}(\mathrm{III})$ and -0.89 for O in $\mathbf{2} ; 1.72$ for $\mathrm{Mn}(\mathrm{IV})$ and -0.66 for O in $\mathbf{3} ; 1.38$ for $\mathrm{Mn}(\mathrm{II}), 1.41$ for $\mathrm{Mn}(\mathrm{III})$, and -1.01 for O in $\mathbf{4 a} ; 1.36$ for $\mathrm{Mn}-$ (II), 1.48 for $\mathrm{Mn}(\mathrm{III})$, and -1.04 for O in $\mathbf{4 b} ; 1.41$ for $\mathrm{Mn}(\mathrm{II})$, 1.57 for $\mathrm{Mn}(\mathrm{IV})$, and -0.90 for O in $5 ; 1.60$ for $\mathrm{Mn}(\mathrm{III}), 1.66$ for $\mathrm{Mn}(\mathrm{IV})$, and -0.74 and -0.78 for O in $\mathbf{6}$, which are much smaller than the formal charges of $2+, 3+, 4+$, and $2-$ for the $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})$, and $\mu$-oxo ions, respectively, indicating that the significant charge delocalization (electron transfer) occurs mainly from di- $\mu$-oxo ions to Mn ions. The differences in atomic charge at the $\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})$, and $\mu$-oxo sites are not as remarkable as expected from their formal charges, and it may be suggested that the influences of the

TABLE 6: Mulliken Gross Orbital Populations in Mn 3d Orbitals of Iso-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the Lowest-Spin Configuration

| complex | 3 d orbital ${ }^{a}$ | Mn1 |  |  |  | 3d orbital ${ }^{a}$ | Mn2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | total | alpha | beta | spin |  | total | alpha | beta | spin |
| 1 | $\mathrm{d}_{x z}$ | 1.064 | 1.003 | 0.061 | 0.942 | $\mathrm{d}_{x z}$ | 1.064 | 0.061 | 1.003 | -0.943 |
|  | $\mathrm{d}_{y z}$ | 1.033 | 1.001 | 0.032 | 0.968 | $\mathrm{d}_{y z}$ | 1.033 | 0.032 | 1.001 | -0.968 |
|  | $\mathrm{d}_{x^{2}}$ | 1.029 | 0.987 | 0.042 | 0.945 | $\mathrm{d}_{x^{2}}$ | 1.030 | 0.042 | 0.987 | -0.945 |
|  | $\mathrm{d}_{x y}$ | 1.118 | 1.001 | 0.117 | 0.884 | $\mathrm{d}_{x y}$ | 1.118 | 0.117 | 1.001 | -0.884 |
|  | $\mathrm{d}_{y^{2}-z^{2}}$ | 1.032 | 0.995 | 0.037 | 0.959 | $\mathrm{d}_{y}{ }^{2}-z^{2}$ | 1.032 | 0.037 | 0.995 | -0.959 |
| 2 | $\mathrm{d}_{x z}$ | 1.100 | 0.996 | 0.104 | 0.892 | $\mathrm{d}_{x z}$ | 1.100 | 0.104 | 0.996 | -0.892 |
|  | $\mathrm{d}_{y z}$ | 1.100 | 1.001 | 0.099 | 0.902 | $\mathrm{d}_{y z}$ | 1.100 | 0.099 | 1.001 | -0.902 |
|  | $\mathrm{d}_{y^{2}}{ }^{2}$ | 1.111 | 0.994 | 0.116 | 0.879 | $\mathrm{d}_{y^{2}}$ | 1.111 | 0.116 | 0.995 | -0.879 |
|  | $\mathrm{d}_{x y}$ | 0.728 | 0.473 | 0.255 | 0.219 | $\mathrm{d}_{x y}$ | 0.728 | 0.255 | 0.473 | -0.219 |
|  | $\mathrm{d}_{x}{ }^{2}-z^{2}$ | 1.101 | 0.988 | 0.113 | 0.874 | $\mathrm{d}_{x}{ }^{2}-z^{2}$ | 1.101 | 0.113 | 0.988 | -0.874 |
| 3 | $\mathrm{d}_{x z}$ | 1.160 | 0.989 | 0.170 | 0.819 | $\mathrm{d}_{x z}$ | 1.160 | 0.170 | 0.989 | -0.819 |
|  | $\mathrm{d}_{y z}$ | 1.194 | 1.002 | 0.192 | 0.810 | $\mathrm{d}_{y z}$ | 1.194 | 0.192 | 1.002 | -0.810 |
|  | $\mathrm{d}_{x^{2}-y^{2}}$ | 0.987 | 0.818 | 0.170 | 0.648 | $\mathrm{d}_{x^{2}-y^{2}}$ | 0.987 | 0.170 | 0.818 | -0.648 |
|  | $\mathrm{d}_{x y}$ | 0.865 | 0.537 | 0.327 | 0.210 | $\mathrm{d}_{x y}$ | 0.865 | 0.327 | 0.537 | -0.210 |
|  | $\mathrm{d}_{2}{ }^{2}$ | 0.782 | 0.569 | 0.212 | 0.357 | $\mathrm{d}_{z}{ }^{2}$ | 0.782 | 0.212 | 0.569 | -0.357 |

${ }^{a}$ First three and last two orbitals correspond to octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively, for each complex.
TABLE 7: Mulliken Gross Orbital Populations in Mn 3d Orbitals of Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the Lowest-Spin Configuration

| complex | 3 d orbital ${ }^{a}$ | Mn1 |  |  |  | 3d orbital ${ }^{a}$ | Mn2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | total | alpha | beta | spin |  | total | alpha | beta | spin |
| 4 a | $\mathrm{d}_{x z}$ | 1.024 | 0.997 | 0.027 | 0.970 | $\mathrm{d}_{x z}$ | 1.144 | 0.142 | 1.002 | -0.860 |
|  | $\mathrm{d}_{y z}$ | 1.028 | 0.999 | 0.028 | 0.971 | $\mathrm{d}_{y z}$ | 1.113 | 0.112 | 1.001 | -0.888 |
|  | $\mathrm{d}_{y^{2}}{ }^{2}$ | 1.033 | 0.996 | 0.037 | 0.960 | $\mathrm{d}_{x^{2}}$ | 1.065 | 0.110 | 0.955 | -0.844 |
|  | $\mathrm{d}_{x y}$ | 1.100 | 1.002 | 0.098 | 0.904 | $\mathrm{d}_{x y}$ | 0.763 | 0.281 | 0.482 | -0.201 |
|  | $\mathrm{d}_{x^{2}-z^{2}}$ | 1.064 | 0.997 | 0.067 | 0.931 | $\mathrm{d}_{y^{2}-z^{2}}$ | 1.109 | 0.113 | 0.996 | -0.882 |
| 4b | $\mathrm{d}_{x z}$ | 1.044 | 0.998 | 0.046 | 0.952 | $\mathrm{d}_{x z}$ | 1.101 | 0.101 | 1.000 | -0.899 |
|  | $\mathrm{d}_{y z}$ | 1.030 | 1.000 | 0.030 | 0.971 | $\mathrm{d}_{\mathrm{yz}}$ | 1.088 | 0.087 | 1.001 | -0.914 |
|  | $\mathrm{d}_{y^{2}}{ }^{2}$ | 1.041 | 0.997 | 0.044 | 0.952 | $\mathrm{d}_{x^{2}}$ | 1.112 | 0.123 | 0.989 | -0.865 |
|  | $\mathrm{d}_{x y}$ | 1.118 | 1.002 | 0.116 | 0.886 | $\mathrm{d}_{x y}$ | 0.733 | 0.262 | 0.471 | -0.210 |
|  | $\mathrm{d}_{x^{2}-z^{2}}$ | 1.037 | 0.990 | 0.047 | 0.942 | $\mathrm{d}_{y^{2}-z^{2}}$ | 1.097 | 0.098 | 1.000 | -0.902 |
| 5 | $\mathrm{d}_{x z}$ | 1.024 | 0.993 | 0.030 | 0.963 | $\mathrm{d}_{x z}$ | 1.269 | 0.269 | 1.000 | $-0.732$ |
|  | $\mathrm{d}_{y z}$ | 1.033 | 1.000 | 0.033 | 0.966 | $\mathrm{d}_{y z}$ | 1.220 | 0.218 | 1.001 | $-0.783$ |
|  | $\mathrm{d}_{x^{2}-y^{2}}$ | 1.035 | 0.995 | 0.040 | 0.954 | $\mathrm{d}_{x^{2}-y^{2}}$ | 1.097 | 0.185 | 0.912 | -0.728 |
|  | $\mathrm{d}_{x y}$ | 1.120 | 1.004 | 0.117 | 0.887 | $\mathrm{d}_{x y}$ | 0.833 | 0.343 | 0.491 | -0.148 |
|  | $\mathrm{d}_{z^{2}}$ | 1.037 | 0.991 | 0.047 | 0.944 | $\mathrm{d}_{z}{ }^{2}$ | 0.710 | 0.256 | 0.454 | -0.198 |
| 6 | $\mathrm{d}_{x z}$ | 1.076 | 0.994 | 0.082 | 0.913 | $\mathrm{d}_{x z}$ | 1.198 | 0.199 | 1.000 | -0.801 |
|  | $\mathrm{d}_{y z}$ | 1.089 | 0.999 | 0.090 | 0.910 | $\mathrm{d}_{y z}$ | 1.209 | 0.207 | 1.001 | -0.794 |
|  | $\mathrm{d}_{\mathrm{x}^{2}}$ | 1.077 | 0.990 | 0.087 | 0.904 | $\mathrm{d}_{x^{2}-y^{2}}$ | 1.032 | 0.209 | 0.823 | -0.614 |
|  | $\mathrm{d}_{x y}$ | 0.753 | 0.510 | 0.243 | 0.266 | $\mathrm{d}_{x y}$ | 0.817 | 0.334 | 0.483 | -0.148 |
|  | $\mathrm{d}_{y^{2}-z^{2}}$ | 1.079 | 0.992 | 0.087 | 0.905 | $\mathrm{d}_{z}{ }^{2}$ | 0.791 | 0.247 | 0.544 | -0.296 |

${ }^{a}$ The first three and last two orbitals correspond to octahedral $t_{2 g}$ and $e_{g}$ orbitals, respectively, for each complex.
electrostatic interactions among the Mn and $\mu$-oxo ions in the different oxidation states on the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ geometries would be rather small. The spin densities are mainly localized on the Mn ions as $\pm 4.75$ for $\mathrm{Mn}(\mathrm{II})$ in $\mathbf{1}, \pm 3.81$ for $\mathrm{Mn}(\mathrm{III})$ in $\mathbf{2}$, $\pm 2.87$ for $\mathrm{Mn}(\mathrm{IV})$ in $\mathbf{3}, 4.76$ for $\mathrm{Mn}(\mathrm{II})$ and -3.80 for Mn (III) in $\mathbf{4 a}$, 4.75 for Mn (II) and -3.85 for Mn (III) in $\mathbf{4 b}, 4.74$ for Mn (II) and -2.62 for $\mathrm{Mn}(\mathrm{IV})$ in $\mathbf{5}$, and 3.94 for $\mathrm{Mn}(\mathrm{III})$ and -2.68 for $\mathrm{Mn}(\mathrm{IV})$ in 6 (lowest-spin configurations), and the $\mu$-oxo bridges have almost zero spin densities in the lowest-spin configurations with the exception of the mixed-valence complexes 5 and $\mathbf{6}$ including the $\mathrm{Mn}(\mathrm{IV})$ ions, whereas the small spin delocaliztion on the $\mu$-oxo ions appears in the highestspin configurations.

The charge densities on $\mathrm{H}_{2} \mathrm{O}$ ligands in the lowest-spin configurations, which are not listed in Table 5, are distributed as follows: -0.04 and -0.05 for $\mathbf{1} ; 0.03$ and 0.09 (axial) and 0.12 (equatoriaal) for $2 ; 0.26$ (axial) and 0.22 (equatorial) for 3; 0.01 and 0.00 (axial to Mn1), -0.02 (between Mn2 and $\mu$-oxo), 0.04 (equatorial to Mn 1 ), and 0.09 (equatorial to Mn2) for $\mathbf{4 a} ;-0.02$ (between Mn1 and $\mu$-oxo), 0.02 (axial to Mn2), 0.05 (equatorial to Mn 1 ), and 0.08 (equatorial to Mn 2 ) for $\mathbf{4 b}$;
0.10 (axial to Mn1), 0.12 (axial to Mn 2 ), 0.08 (equatorial to Mn 1 ), and 0.11 (equatorial to Mn 2 ) for $\mathbf{5 ;} 0.13$ (axial to Mn 1 ), 0.17 (axial to Mn2), 0.15 and 0.16 (equatorial to Mn1), and 0.17 and 0.18 (equatorial to Mn 2 ) for 6 . Relatively larger positive charges are found only for $\mathbf{3}$ with the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}-$ (IV) oxidation state, and it is thus likely that the axial $\mathrm{H}_{2} \mathrm{O}$ molecules on each Mn ion in $\mathbf{3}$ are apart from each other without hydrogen bonds as shown in Figure 4c.
The electron populations for majority-spin components of oxidized $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$ ions indicate that the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ orbitals are always fully occupied with the populations near 1.0 , and the oxidized electrons are removed from the octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals in which the primary and secondary oxidations from Mn (II) to Mn (III) and from Mn (III) to Mn (IV) occur in the equatorial $\left(\mathrm{d}_{x y}\right)$ and axial $\left(\mathrm{d}_{z^{2}}\right)$ orbitals, respectively. Concerning the $\mathrm{Mn}(\mathrm{IV})$ oxidation state, the $\mathrm{d}_{x^{2}-y^{2}}$ populations of $\mathbf{3}$ and $\mathbf{6}$ indicate slightly lower values of about 0.82 for majority spin because of the mixing with the $\mathrm{d}_{z^{2}}$ orbital at the same Mn site. The nonzero populations for minority-spin components, in contrast to the formal oxidation states with zero populations, would be the contributions of the partial electron transfer of

TABLE 8: Mn-Based Natural Orbitals of Iso-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the Lowest-Spin Configuration

| natural orbital | 1 |  |  | 2 |  |  | 3 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n^{a}$ | Mn1 | Mn2 | $n^{a}$ | Mn1 | Mn2 | $n^{a}$ | Mn1 | Mn2 |
| LUMO+3 |  |  |  |  |  |  | $0.008^{\text {b }}$ | + $\mathrm{d}^{2}$ | $-\mathrm{d}_{2}{ }^{2}$ |
| LUMO+2 |  |  |  |  |  |  | $0.011{ }^{\text {b }}$ | $+\mathrm{d}^{2}{ }^{2}$ | $+\mathrm{d}^{2}$ |
| LUMO+1 |  |  |  | $0.023{ }^{\text {b }}$ | $+\mathrm{d}_{x y}$ | $-\mathrm{d}_{x y}$ | $0.019^{\text {b }}$ | $+\mathrm{d}_{x y}$ | $-\mathrm{d}_{x y}$ |
| LUMO |  |  |  | $0.038^{\text {b }}$ | $+\mathrm{d}_{x y}$ | + $\mathrm{d}_{x y}$ | $0.036^{\text {b }}$ | $+\mathrm{d}_{x y}$ | +d $\mathrm{d}_{x y}$ |
| SOMO+5 | $0.900^{\text {c }}$ | $+\mathrm{d}_{x}{ }^{2}$ | $-\mathrm{d}_{x^{2}}$ |  |  |  |  |  |  |
| SOMO+4 | $0.932^{\text {b }}$ | $+\mathrm{d}_{x y}$ | $-\mathrm{d}_{x y}$ | $0.792^{\text {c }}$ | $+\mathrm{d}_{x z}$ | $+\mathrm{d}_{x z}$ |  |  |  |
| SOMO+3 | $0.974{ }^{\text {c }}$ | $+\mathrm{d}_{x z}$ | $+\mathrm{d}_{x z}$ | $0.813^{b}$ | $+\mathrm{d}_{x^{2}-z^{2}}$ | $-\mathrm{d}_{x^{2}-z^{2}}$ | $0.761{ }^{\text {c }}$ | + $\mathrm{d}_{y z}$ | $-\mathrm{d}_{\mathrm{yz}}$ |
| SOMO+2 | $0.993{ }^{\text {b }}$ | + $\mathrm{d}_{y^{2}-z^{2}}$ | + $\mathrm{d}_{y^{2}-z^{2}}$ | $0.858^{c}$ | $+\mathrm{d}_{\mathrm{yz}}$ | $-\mathrm{d}_{y z}$ | $0.761{ }^{\text {c }}$ | $+\mathrm{d}_{x z}$ | $+\mathrm{d}_{x z}$ |
| SOMO+1 | $0.998^{\text {c }}$ | + $\mathrm{d}_{\mathrm{yz}}$ | $+\mathrm{d}_{y z}$ | $0.861{ }^{\text {c }}$ | $+\mathrm{d}_{y^{2}}$ | $+\mathrm{d}_{y^{2}}$ | $0.867^{\text {c }}$ | $+\mathrm{d}_{x^{2}-y^{2}}$ | + $\mathrm{d}_{x^{2}-y^{2}}$ |
| SOMO-1 | $1.002^{c}$ | $+\mathrm{d}_{y z}$ | $-\mathrm{d}_{y z}$ | $1.139^{c}$ | $+\mathrm{d}_{y^{2}}$ | $-\mathrm{d}_{y^{2}}$ | $1.133^{c}$ | $+\mathrm{d}^{2}-y^{2}$ | $-\mathrm{d}^{2}-y^{2}$ |
| SOMO-2 | $1.007^{\text {b }}$ | $+\mathrm{d}_{y^{2}-z^{2}}$ | $-\mathrm{d}_{y^{2}-z^{2}}$ | $1.142^{c}$ | $+\mathrm{d}_{y z}$ | + $\mathrm{d}_{y z}$ | $1.239^{c}$ | $+\mathrm{d}_{x z}$ | $-\mathrm{d}_{x z}$ |
| SOMO-3 | $1.026^{\text {c }}$ | $+\mathrm{d}_{x z}$ | $-\mathrm{d}^{x z}$ | $1.187^{\text {b }}$ | $+\mathrm{d}_{x^{2}-z^{2}}$ | $+\mathrm{d}_{x^{2}-z^{2}}$ | $1.239^{c}$ | $+\mathrm{d}_{y z}$ | $+\mathrm{d}_{y z}$ |
| SOMO-4 | $1.068^{\text {b }}$ | $+\mathrm{d}_{x y}$ | $+\mathrm{d}_{x y}$ | $1.208^{c}$ | + $\mathrm{d}_{x z}$ | $-\mathrm{d}_{x z}$ |  |  |  |
| SOMO-5 | $1.100^{c}$ | $+\mathrm{d}_{x^{2}}$ | + $\mathrm{d}^{2}$ |  |  |  |  |  |  |

${ }^{a} n$ indicates the electron occupation number. ${ }^{b}$ Specified orbitals correspond to octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals. ${ }^{c}$ Specified orbitals correspond to octahedral $\mathrm{t}_{2 \mathrm{~g}}$ orbitals.

TABLE 9: Mn-Based Natural Orbitals of Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the Lowest-Spin Configuration

| natural orbital | 4 a |  |  | 4b |  |  | 5 |  |  | 6 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n^{a}$ | Mn1 | Mn2 | $n^{a}$ | Mn1 | Mn2 | $n^{a}$ | Mn1 | Mn2 | $n^{a}$ | Mn1 | Mn2 |
| LUMO+2 |  |  |  |  |  |  |  |  |  | $0.007{ }^{\text {b }}$ |  | $+\mathrm{d}_{z}{ }^{2}$ |
| LUMO+1 |  |  |  |  |  |  | $0.010^{\text {b }}$ |  | $+\mathrm{d}_{z^{2}}$ | $0.012^{\text {b }}$ |  | $+\mathrm{d}_{x y}$ |
| LUMO | $0.024^{\text {b }}$ |  | $+\mathrm{d}_{x y}$ | $0.027^{\text {b }}$ |  | + $\mathrm{d}_{x y}$ | $0.013{ }^{\text {b }}$ |  | $+\mathrm{d}_{x y}$ | $0.048^{\text {b }}$ | $+\mathrm{d}_{x y}$ |  |
| SOMO+4 | $0.869^{\text {b,c }}$ | $+\mathrm{d}_{x^{2}-z^{2}}$ | $-\mathrm{d}^{2}$ | $0.874{ }^{\text {b,c }}$ | $+\mathrm{d}_{x^{2}-z^{2}}$ | $-\mathrm{d}_{x^{2}}$ |  |  |  |  |  |  |
| SOMO+3 | $0.898^{\text {c }}$ | $+\mathrm{d}_{x z}$ | $+\mathrm{d}_{x z}$ | $0.923{ }^{\text {c }}$ | $+\mathrm{d}_{x z}$ | + $\mathrm{d}_{x z}$ | $0.835^{\text {c }}$ | $+\mathrm{d}_{x z}$ | $+\mathrm{d}_{x z}$ | $0.780^{c}$ | $+\mathrm{d}_{x z}$ | $+\mathrm{d}_{x z}$ |
| SOMO+2 | $0.936^{c}$ | $+\mathrm{d}_{y z}$ | $-\mathrm{d}_{y z}$ | $0.940^{\text {b,c }}$ | + $\mathrm{d}_{y^{2}}$ | + $\mathrm{d}_{y^{2}-z^{2}}$ | $0.860^{b, c}$ | $+\mathrm{d}_{z^{2}}$ | $+\mathrm{d}_{x^{2}-y^{2}}$ | $0.797^{\text {b,c }}$ | + $\mathrm{d}_{y^{2}-z^{2}}$ | $-\mathrm{d}_{x^{2}-y^{2}}$ |
| SOMO+1 | $0.944^{b, c}$ | $+\mathrm{d}_{y^{2}}$ | $+\mathrm{d}_{y^{2}-z^{2}}$ | $0.951{ }^{\text {c }}$ | $+\mathrm{d}_{y z}$ | $-\mathrm{d}_{y z}$ | $0.885^{\text {c }}$ | $+\mathrm{d}_{y z}$ | $-\mathrm{d}_{y z}$ | $0.799^{c}$ | $+\mathrm{d}_{\mathrm{yz}}$ | $-\mathrm{d}_{y z}$ |
| SOMO2 |  |  |  |  |  |  | $1.0^{b, c}$ | $+\mathrm{d}_{x y}-\mathrm{d}_{x^{2}-y^{2}}$ |  |  |  |  |
| SOMO1 | $1.0^{\text {b }}$ | + $\mathrm{d}_{x y}$ |  | $1.0^{\text {b }}$ | + $\mathrm{d}_{x y}$ |  | $1.0^{b, c}$ | + $\mathrm{d}_{x^{2}-y^{2}}+\mathrm{d}_{x y}$ |  | $1.0^{b, c}$ | $+\mathrm{d} x^{2}-\mathrm{d}_{y^{2}-z^{2}}$ |  |
| SOMO-1 | $1.056^{\text {b,c }}$ | $+\mathrm{d}_{y^{2}}$ | $-\mathrm{d}_{y^{2}-z^{2}}$ | $1.049^{c}$ | + $\mathrm{d}_{y z}$ | + $\mathrm{d}_{y z}$ | $1.115^{c}$ | $+\mathrm{d}_{y z}$ | + $\mathrm{d}_{y z}$ | $1.201^{c}$ | + $\mathrm{d}_{y z}$ | $+\mathrm{d}_{y z}$ |
| SOMO-2 | $1.064{ }^{c}$ | $+\mathrm{d}_{y z}$ | $+\mathrm{d}_{y z}$ | $1.060^{b, c}$ | $+\mathrm{d}_{y^{2}}$ | $-\mathrm{d}_{y^{2}-z^{2}}$ | $1.140^{b, c}$ | $+\mathrm{d}_{2}{ }^{2}$ | $-\mathrm{d}_{x^{2}-y^{2}}$ | $1.203^{b, c}$ | $+\mathrm{d}_{y^{2}-z^{2}}$ | + $\mathrm{d}_{x^{2}-y^{2}}$ |
| SOMO-3 | $1.102^{c}$ | $+\mathrm{d}_{x z}$ | $-\mathrm{d}_{x z}$ | $1.077^{c}$ | + $\mathrm{d}_{x z}$ | $-\mathrm{d}_{x z}$ | $1.165^{c}$ | $+\mathrm{d}_{x z}$ | $-\mathrm{d}_{x z}$ | $1.221^{c}$ | $+\mathrm{d}_{x z}$ | $-\mathrm{d}_{x z}$ |
| SOMO-4 | $1.131^{b, c}$ | $+\mathrm{d}_{x^{2}-z^{2}}$ | $+\mathrm{d}^{2}$ | $1.126^{\text {b,c }}$ | $+\mathrm{d}^{2}-z^{2}$ | $+\mathrm{d}_{x^{2}}$ |  |  |  |  |  |  |

${ }^{a}{ }_{n}$ indicates the electron occupation number. ${ }^{b}$ Specified orbitals correspond to octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals. ${ }^{c}$ Specified orbitals correspond to octahedral $\mathrm{t}_{2 \mathrm{~g}}$ orbitals.
doubly occupied electrons from the ligand to Mn , suggesting superexchange interactions through the di- $\mu$-oxo bridge. Such covalent mixings are smaller for the $\mathrm{Mn}(\mathrm{II})$ ion than for Mn (III) and $\mathrm{Mn}(\mathrm{IV})$ ions because the Mn (II) ion has longer separations from ligands. The electron populations for minorityspin components in the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals of $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$ ions are respectively given as follows: $0.10,0.10$, and $0.12\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.26 and $0.11\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}(\mathrm{III})$ in $2 ; 0.17$, 0.19 , and $0.17\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.33 and $0.21\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}(\mathrm{IV})$ in 3 ; $0.14,0.11$, and $0.11\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.28 and $0.11\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}(\mathrm{III})$ in 4a; 0.10, 0.09, and $0.12\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.26 and $0.10\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}(\mathrm{III})$ in 4b; 0.27, 0.22, and $0.19\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.34 and $0.26\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}-$ (IV) in 5; 0.08, 0.09, and $0.09\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.24 and $0.09\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}(\mathrm{III})$ in 6; 0.20, 0.21 , and $0.21\left(\mathrm{t}_{2 \mathrm{~g}}\right)$ and 0.33 and $0.25\left(\mathrm{e}_{\mathrm{g}}\right)$ for $\mathrm{Mn}(\mathrm{IV})$ in $\mathbf{6}$. These results indicate that the larger mixing arises for the $\mathrm{d}_{x y}$ component of octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals which are extended directly toward $\mu$-oxo ions.

The Mulliken population analyses of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}$ (IV) oxidation states have been reported in ref 30. The spin densities of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in these oxidation states listed in Table 5 are consistent with those of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ as $\pm 3.69$ for Mn(III) in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}$ (III) dimer, $\pm 2.78$ for Mn (IV) in the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ dimer, and 3.71 for Mn (III) and -2.50 for Mn (IV) in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}$ (IV) dimer, although the absolute values are somewhat smaller in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ than in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$. The electron populations for majority- and minority-spin components of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ were given as
follows: $0.99-1.00$ and $0.14-0.15\left(\mathrm{t}_{2 \mathrm{~g}}\right), 0.47$ and $0.29\left(\mathrm{~d}_{x y}\right)$, and 0.99 and $0.17\left(\mathrm{~d}_{z^{2}}\right)$ for $\mathrm{Mn}(\mathrm{III})$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ center; 0.99 and $0.13-0.24\left(\mathrm{t}_{2 \mathrm{~g}}\right), 0.51$ and $0.37\left(\mathrm{~d}_{x y}\right)$, and 0.49 and $0.35\left(\mathrm{~d}_{z^{2}}\right)$ for $\mathrm{Mn}(\mathrm{IV})$ in the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ center; $0.99-$ 1.00 and $0.09-0.13\left(\mathrm{t}_{2 \mathrm{~g}}\right), 0.50$ and $0.30\left(\mathrm{~d}_{x y}\right), 0.95$ and 0.21 $\left(\mathrm{d}_{z^{2}}\right)$ for $\mathrm{Mn}(\mathrm{III})$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ center; $0.99-1.00$ and $0.25-0.26\left(\mathrm{t}_{2 \mathrm{~g}}\right), 0.47$ and $0.36\left(\mathrm{~d}_{x y}\right)$, and 0.43 and $\left.0.34\left(\mathrm{~d}_{z}\right)^{2}\right)$ for $\mathrm{Mn}(\mathrm{IV})$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ center. In comparison with these gross orbital populations of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$, the electron populations for minority-spin components seem to become slightly smaller in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{9+}$ as shown in Tables 6 and 7, particularly for the octahedral $\mathrm{e}_{\mathrm{g}}$ orbitals in the axial direction of the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ complex, suggesting that the orbital mixings between Mn and $\mu$-oxo ions are smaller in the $\mathrm{H}_{2} \mathrm{O}$ coordinated complexes than in the $\mathrm{NH}_{3}$-coordinated complexes.

## VII. Natural Orbital Analysis of the Model Complex

We performed the natural orbital analysis of the model complexes to elucidate the singly occupied electron configurations of the $\mathrm{Mn}-\mathrm{Mn}$ center in each oxidation state. The Mn based natural orbitals of iso- and mixed-valence complexes in the lowest-spin configuration are listed in Tables 8 and 9, respectively, in which the main 3d orbitals with the largest components in Mulliken composition are given together with the electron occupation numbers. Figures 8 and 9 , respectively, show the graphical presentations of the octahedral $\mathrm{e}_{\mathrm{g}}$ natural orbitals of the iso- and mixed-valence complexes in the lowest-


Figure 8. Octahedral $\mathrm{e}_{\mathrm{g}}$ natural orbitals of iso-valence complexes in the lowest-spin state: (a) $\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}$ (1), (b) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}(\mathbf{2})$, and (c) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{4+}(3)$. Isocontours correspond to 0.025 au .


Figure 9. Octahedral $e_{g}$ natural orbitals of mixed-valence complexes in the lowest-spin state: $(a)\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}(\mathbf{4 a}),(\mathbf{b})\left[\mathbf{M n} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}(\mathbf{4 b})$, (c) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}(5)$, and (d) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}(6)$. Isocontours correspond to 0.025 au .
spin configuration, and it is confirmed from these Figures that the type of octahedral $\mathrm{e}_{\mathrm{g}}$ orbital in the axial direction for each Mn site changes depending on the oxidation state of the MnMn center as mentioned in the preceding section.

The pairs of Mn -based natural orbitals denoted as $\mathrm{SOMO}+m$ and SOMO $-m$, which give 2.0 as the sum of their electron occupation numbers, are composed of the in-phase and out-ofphase combinations between 3d orbitals centered at each Mn site, and these pairs form the broken-symmetry solutions of singlet-coupled biradicals. The pairings of $\mathrm{d}_{x z}$ and $\mathrm{d}_{y z}$ orbitals are common to $\mathbf{1 - 6}$. The 3d components of SOMO1, SOMO2, LUMO, LUMO +1 , and LUMO +2 for $\mathbf{4 - 6}$ are well localized at one of the Mn sites (Mn1 or Mn2). The SOMO1 and SOMO2 are the pure singly occupied orbitals with an electron occupation number of 1.0 , and the SOMO for $\mathbf{4 a}$ and $\mathbf{4 b}$ is a nearly single 3 d orbital of the $\mathrm{Mn}(\mathrm{II})$ site, whereas the SOMO for 5 and $\mathbf{6}$ has largely mixed 3d components within the same center of the Mn (II) or Mn (III) site (the SOMO1 and SOMO2 for $\mathbf{5}$ are the degenerate natural orbitals with $n=1.0$ ).

The electron occupations of the octahedral $\mathrm{e}_{\mathrm{g}}$ natural orbitals shown in Tables 8 and 9 change depending on the oxidation of the $\mathrm{Mn}-\mathrm{Mn}$ center as follows: occupied $\mathrm{d}_{x y}(\mathrm{SOMO} \pm 4)$ and occupied $\mathrm{d}_{y^{2}-z^{2}}(\mathrm{SOMO} \pm 2)$ for $\mathbf{1}$ with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II})$; occupied $\mathrm{d}_{x^{2}-z^{2}}(\mathrm{SOMO} \pm 3)$ and unoccupied $\mathrm{d}_{x y}$ (LUMO and LUMO+1) for 2 with $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$; unoccupied $\mathrm{d}_{x y}$ (LUMO and LUMO +1 ) and unoccupied $\mathrm{d}_{2}{ }^{2}(\mathrm{LUMO}+2$ and LUMO+3) for 3 with $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$; occupied $\mathrm{d}_{x^{2}-z^{2}}$ on $\mathrm{Mn} 1(\mathrm{SOMO} \pm 4)$,
occupied $\mathrm{d}_{x y}$ on Mn1 (SOMO), occupied $\mathrm{d}_{y^{2}-z^{2}}$ on Mn2 ( $\mathrm{SOMO} \pm 1$ for $\mathbf{4 a}$ and $\mathrm{SOMO} \pm 2$ for $\mathbf{4 b}$ ), and unoccupied $\mathrm{d}_{x y}$ on Mn2 (LUMO) for 4 with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$; occupied $\mathrm{d}_{z^{2}}$ on Mn1 (SOMO $\pm 2$ ), occupied $\mathrm{d}_{x y}$ on Mn1 (SOMO1 and SOMO2), unoccupied $\mathrm{d}_{x y}$ on Mn2 (LUMO), and unoccupied $\mathrm{d}_{z^{2}}$ on Mn2 (LUMO+1) for 5 with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$; occupied $\mathrm{d}_{y^{2}}{ }^{2} z^{2}$ on Mn1 ( $\mathrm{SOMO} \pm 2$ ), unoccupied $\mathrm{d}_{x y}$ on Mn1 (LUMO), unoccupied $\mathrm{d}_{x y}$ on Mn2 (LUMO+1), and unoccupied $\mathrm{d}_{z^{2}}$ on Mn2 (LUMO+2) for $\mathbf{6}$ with $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$. However, the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ orbitals are always occupied in $\mathbf{1 - 6}$. It is consequently confirmed that the one-electron oxidations of the Mn ion from $\mathrm{Mn}(\mathrm{II})$ to Mn (III) and from Mn (III) to Mn (IV) occur by removing occupied $\mathrm{e}_{\mathrm{g}}$ electrons in the equatorial $\left(\mathrm{d}_{x y}\right)$ and axial $\left(\mathrm{d}_{z^{2}}\right)$ directions, respectively.

As seen in Figures 8 and 9, the equatorial $\mathrm{e}_{\mathrm{g}}$ orbitals (SOMO $\pm 4$ in Figure 8a, LUMO and LUMO +1 in Figures 8 b and 8c, SOMO and LUMO in Figures 9a and 9b, SOMO1, SOMO2, and LUMO in Figure 9c, and LUMO and LUMO+1 in Figure 9d) indicate the antibonding character between Mn and $\mu$-oxo ions and between the Mn ion and the equatorial $\mathrm{H}_{2} \mathrm{O}$ ligand, whereas the axial $\mathrm{e}_{\mathrm{g}}$ orbitals $(\mathrm{SOMO} \pm 2$ in Figure 8a, $S O M O \pm 3$ in Figure 8b, LUMO+2 and LUMO+3 in Figure 8c, $\mathrm{SOMO} \pm 4$ in Figure 9a, $\mathrm{SOMO} \pm 2$ in Figure 9b, $\mathrm{SOMO} \pm 2$ and LUMO +1 in Figure 9c, and SOMO, $\mathrm{SOMO} \pm 2$, and LUMO +2 in Figure 9d) indicate the antibonding nature between the Mn ion and the axial $\mathrm{H}_{2} \mathrm{O}$ ligand. Because the electron configurations of these orbitals vary according to the oxidation

TABLE 10: Experimental and Theoretical Effective Exchange Integrals of Iso- and Mixed-Valence $\mathbf{M n}_{2} \mathbf{O}_{2}$ Complexes

| oxidation state | $-J^{a}$ | ref | oxidation state | $-J^{a}$ | $\|B\|^{b}$ | ref |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | experiment | experiment |  |  |  |  |
| $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ | 86 | 30 | $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ | 220 |  | 29 |
|  | 87, 101 | 34 |  | 119-159 |  | 30 |
| $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ | 120 | 29 |  | 101-220 |  | 34 |
|  | 126, 144 | 30 |  |  |  |  |
|  | 40-147 | 34 |  |  |  |  |
|  | 78-188 | 54 |  |  |  |  |
|  | calculation |  |  | calculation |  |  |
| Mn (II) -Mn (II) | 2 | this work | Mn (II) -Mn (III) | 341, 337 | 293 | this work |
| $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ | 154 | this work |  | 23 |  |  |
|  | 172 | 30 | $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ | 1145 | 1821 | this work <br> 29 |
|  | 176 | 33 |  | 405 |  |  |
|  | 89 | 34 |  | 391 | 143 | 30 |
| $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ | 106 | this work |  | 268 | 135 | 33 |
|  | 232 | 29 |  | 118 |  | 34 |
|  | 274 | 30 |  |  |  |  |
|  | 125 | 31 |  |  |  |  |
|  | 118 | 32 |  |  |  |  |
|  | 99 | 32 |  |  |  |  |
|  | 149 | 32 |  |  |  |  |
|  | 129 | 33 |  |  |  |  |
|  | 101 | 34 |  |  |  |  |
|  | 124 | 53 |  |  |  |  |
|  | 115 | 53 |  |  |  |  |

${ }^{a}$ Effective exchange integrals are given in $\mathrm{cm}^{-1} .{ }^{b}$ Resonance delocalization parameters are given in $\mathrm{cm}^{-1}$.
states of complexes as described above such that both orbitals are occupied for $\mathrm{Mn}(\mathrm{II})$, the equatorial and axial orbitals are unoccupied and occupied for $\mathrm{Mn}(\mathrm{III})$, and both orbitals are unoccupied for $\mathrm{Mn}(\mathrm{IV})$, the deformations of molecular geometry with the oxidation of the $\mathrm{Mn}-\mathrm{Mn}$ center would be induced for the Mn -ligand distances, which are essentially expected as Mn (II) $-\mathrm{O}>\mathrm{Mn}$ (III) -O and Mn (IV) -O for the $\mu$-oxo bridge, Mn (II) $-\mathrm{OH}_{2}>\mathrm{Mn}(\mathrm{III})-\mathrm{OH}_{2}$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{OH}_{2}$ for equatorial ligands, and $\mathrm{Mn}(\mathrm{II})-\mathrm{OH}_{2}$ and $\mathrm{Mn}(\mathrm{III})-\mathrm{OH}_{2}>\mathrm{Mn}(\mathrm{IV})-\mathrm{OH}_{2}$ for axial ligands, corresponding to the optimized structures given in section V.

## VIII. Magnetic Interaction of Model Complex

We summarize the experimental and theoretical $J$ values of the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ complexes with the di- $\mu$-oxo bridge available from refs $29-34,53$, and 54 in Table 10 together with the $J$ values of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ calculated in this work. The experimental values were measured for the complexes with various ligands (mainly N -donor). The theoretical values were calculated by the broken-symmetry DFT method for the following complexes: $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}(\mathrm{OAc})(\mathrm{TACN})_{2}\right]^{2+}$ and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mu-\mathrm{O}_{2}\right)(\mathrm{TACN})_{2}\right]^{2+}$ in ref 29; $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ in ref 30; $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mu-\mathrm{O}_{2}\right)\left(\mathrm{NH}_{3}\right)_{6}\right]^{2+}$ in ref 31; $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}(\text { pic })_{4}\right],\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{4+}$, and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2^{-}}\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{4+}$ in ref 32; $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{O}_{2} \mathrm{CH}\right)\left(\mathrm{NH}_{3}\right)_{6}\right]^{9+}$ in ref 33; $\left[\mathrm{Mn}_{2} \mathrm{O}_{2^{-}}\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{6}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}\right]^{q+}$ in ref 34; $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NHCHCO}_{2}\right)_{4}\right]$ in ref 53. In our estimation of the $J$ values for $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$, the isovalence centers of $\mathrm{Mn}($ II $)-\mathrm{Mn}(\mathrm{II}), \mathrm{Mn}(\mathrm{III})-\mathrm{Mn}($ III $)$, and $\mathrm{Mn}-$ (IV) $-\mathrm{Mn}(\mathrm{IV})$ and the mixed-valence centers of Mn (II) $-\mathrm{Mn}($ III $)$ and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ were examined, and the optimized structures of the highest- and lowest-spin configurations were used to obtain the $J$ values as in ref 30 . We performed the geometry optimizations with some of the geometrical constraints for the mixed-valence complexes 4 and $\mathbf{6}$ in the delocalized highest-spin state to take account of the delocalization effects for the $J$ values (eq 8) in which an extra electron was placed in the in-phase or out-of-phase combinations of the $\mathrm{d}_{x y}$ orbitals on each Mn site for $\mathbf{4}$ and in those of the $\mathrm{d}_{z}{ }^{2}$ orbitals on each Mn site for $\mathbf{6}$, respectively. The calculated results of energies, geometries, and charge and spin densities of mixed-valence
complexes in the delocalized highest-spin states are given in Appendix B.

As described in refs 49,50 , and 53 as examples, the theoretical $J$ values largely depend on the computational methods such as the combination of the exchange and correlation functionals employed in DFT calculation. Thus, the $J$ values of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the iso-valence oxidation state $(\mathbf{1}-\mathbf{3})$ listed in Table 10 would indicate that our results are consistent with the other theoretical results in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ and Mn (IV) $-\mathrm{Mn}(\mathrm{IV})$ oxidation states. In ref 32 , it was shown that the theoretical $-J$ value of the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ complex increases with ligands in the order $\mathrm{H}_{2} \mathrm{O}\left(-J=99 \mathrm{~cm}^{-1}\right)<$ pic $(-J=$ $\left.118 \mathrm{~cm}^{-1}\right)<\mathrm{NH}_{3}\left(-J=149 \mathrm{~cm}^{-1}\right)$ in proportion to the number of coordinating N atoms, which is in accordance with the experimental tendency. The results obtained in this study therefore are consistent with the results of ref 32. Although the experimental and theoretical values of the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II})$ complex have not been reported, the $J$ value calculated in this work is much smaller for $\mathbf{1}$ than for $\mathbf{2}$ and $\mathbf{3}$, which would be caused by longer $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II})$ and $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ separations from a geometrical viewpoint, suggesting weak magnetic interactions.

The experimental results seem to show the relation of $-J$ for $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})<-J$ for $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ in contrast to our result of $-J$ for $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})=154 \mathrm{~cm}^{-1}>-J$ for $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})=106 \mathrm{~cm}^{-1}$. However, it is noted that the experimental $J$ values of the $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ complexes summarized in refs 34 and 54 indicate the following distribution of reported 28 and 20 data: 19 (ref 34) and 10 (ref 54) compounds in $-J<100 \mathrm{~cm}^{-1}$ and 9 (ref 34) and 10 (ref 54) compounds in $100 \mathrm{~cm}^{-1} \leq-J$, whereas data of only three compounds have been given in refs 30 and 34 for the experimental $J$ values of the $\mathrm{Mn}($ III $)-\mathrm{Mn}(\mathrm{III})$ complexes as $-J$ $\approx 90-100 \mathrm{~cm}^{-1}$. It is, thus, impossible to conclude exactly the order of experimental $J$ values with overlap in the $\mathrm{Mn}(\mathrm{III})-$ $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ oxidation states.

As for the mixed-valence complexes, Table 10 indicates that the $-J$ values are much larger in our calculations than in other theoretical and experimental work. (It is noted that the $J$ values
of the mixed-valence complexes reported in ref 34 may not include resonance delocalization effects.) However, it has been pointed out in refs 29,48 , and 52 that when the trapping energy $\Delta E_{\text {trap }}$ largely exceeds the resonance energy, $\Delta E_{\text {trap }} \gg B(S+$ $1 / 2)$, the observed $J$ value would be given by $J_{\text {eff }}=J+\left(B^{2} /\right.$ $\Delta E_{\text {trap }}$ ) giving a positive (ferromagnetic) shift due to the partial delocalization, and the trapping energy is the same as the energy of the intervalence charge-transfer optical band, $\Delta E_{\mathrm{op}}=\Delta E_{\text {trap }}$. Experimentally, ${ }^{55-57} \Delta E_{\text {op }}$ has been estimated as $7124 \mathrm{~cm}^{-1}$ for $\left[\mathrm{Mn}(\mathrm{III}) \mathrm{Mn}(\mathrm{IV})(\mu-\mathrm{O})(\mathrm{OAc})_{2}(\mathrm{TACN})_{2}\right]^{3+}$ and $12000 \mathrm{~cm}^{-1}$ for $\left[\mathrm{Mn}(\mathrm{III}) \mathrm{Mn}(\mathrm{IV}) \mathrm{O}_{2} \mathrm{~L}_{4}\right]^{3+}\left(L=2,2^{\prime}\right.$-bipyridine or 1,10-phenanthroline). We roughly estimate the $B^{2} / \Delta E_{\text {trap }}$ value for $[\mathrm{Mn}(\mathrm{IIII})-$ $\left.\mathrm{Mn}(\mathrm{IV}) \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}$ assuming that $\Delta E_{\text {trap }} \approx 10000 \mathrm{~cm}^{-1}$ to obtain a value of $332 \mathrm{~cm}^{-1}$. (We note that the value of $9 \mathrm{~cm}^{-1}$ is obtained for $\left[\mathrm{Mn}(\mathrm{II}) \mathrm{Mn}(\mathrm{III}) \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}$with the same assumption of $\Delta E_{\text {trap }} \approx 10000 \mathrm{~cm}^{-1}$ ). The $J$ values of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{9^{+}}$in the mixed-valence oxidation states (4 and 6) listed in Table 10 may be overestimated with uncertainties also because eq 8 is used for the fully delocalized limit of symmetric high-spin states, as discussed in ref 30 . Moreover, the hydrogen bonds are formed in the broken-symmetry states of $\mathbf{4}$ and $\mathbf{6}$ as shown in Figures 5a, 5b, and 5d, whereas the hydrogen bonds in the delocalized high-spin states of $\mathbf{4}$ and $\mathbf{6}$ vanish or become weak with the longer bond lengths of $2.0-2.1 \AA$, indicating that the broken-symmetry states would be more stabilized by the hydrogen bonds that lead to large energy gaps between the highest- and lowest-spin configurations in the estimation of $J$ values. In the case when the lower energy of delocalized highestspin states is applied to eq 8 instead of the averaged energy of delocalized highest-spin states as the localized limit, ${ }^{29,33}$ we obtain $-J$ values of $269 \mathrm{~cm}^{-1}$ for $\mathbf{4 a}, 264 \mathrm{~cm}^{-1}$ for $\mathbf{4 b}$, and $555 \mathrm{~cm}^{-1}$ for 6 . However, if the resonance delocalization of the second term in eq 5 is ignored, and the localized highestspin state of eq 4 is adopted for to the calculation of $J$ values, the $-J$ values become $11 \mathrm{~cm}^{-1}$ for $\mathbf{4 a}, 10 \mathrm{~cm}^{-1}$ for $\mathbf{4 b}$, and $163 \mathrm{~cm}^{-1}$ for $\mathbf{6}$. The result for the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}$ (IV) complex 6 obtained from eq 4 seems to show similarity to the experimental values, suggesting a valence-trapped nature.

Here, we briefly discuss the superexchange pathways between the Mn sites through the di- $\mu$-oxo bridge on the basis of the magnetic orbitals, which are obtained using eq 12 from the singlet-coupled natural orbitals of the broken-symmetry solutions. Figures 10 and 11 give graphical presentations of the singly occupied magnetic orbitals of the iso- and mixed-valence complexes, respectively, and the compositions of these orbitals are given in Appendix C. Although the compositions of the unrestricted canonical orbitals of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ with the Mn(III) $-\mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ centers were reported in ref 30 , it may be impossible to compare the canonical orbitals directly with the magnetic orbitals. Because some of the 3d components of the Mn ions would be distributed over several canonical orbitals in a delocalized nature, it may be difficult to regard the specific canonical orbitals as singly occupied Mn 3d orbitals that characterize the magnetic properties of the molecule. It may therefore be reasonable to use the well-defined singly occupied orbitals such as the natural orbitals or the magnetic orbitals in the analysis of magnetic properties. ${ }^{49-51,53}$

Considering the interaction between Mn (parallel unpaired spins) and $\mu$-oxo (closed shell) sites in the formal oxidation state, the contribution from the electron configuration, in which an electron of the $\mu$-oxo ion with antiparallel spin to the electrons of the Mn ion that are transferred to one of the Mn ions, arises in the interacting system. The remaining spin on the $\mu$-oxo
bridge would induce a spin polarization with opposite direction on another Mn center. This leads to the antiparallel spin alignment between two Mn ions if another Mn ion has overlap interactions with the $\mu$-oxo ions (superexchange interaction). Consequently, both orbital mixing between one Mn center and a di- $\mu$-oxo bridge and the nonzero orbital overlap between the di- $\mu$-oxo bridge and another Mn center are crucial for effective magnetic coupling by the superexchange mechanism.

The mixing weights of the di- $\mu$-oxo bridge in the magnetic orbitals are smaller for the $\mathrm{Mn}(\mathrm{II})$ ion than for the $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$ ions with the exception of the $\mathrm{d}_{x y}$ orbital (Appendix C) as expected from the longer $\mathrm{Mn}-\mathrm{O}$ distances for the Mn (II) site than for the $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$ sites. Because the $\mathrm{d}_{x y}$ orbital on the $\mathrm{Mn}(\mathrm{II})$ site, which faces the $\mu$-oxo ion, interacts with the $\mathrm{p}_{\sigma}$ orbital of the $\mu$-oxo bridge ( $\mathrm{SOMO} \pm 4$ in Figure 10a for 1, SOMO in Figures 9a and $9 b$ for $\mathbf{4 a}$ and $\mathbf{4 b}$, and SOMO1 and SOMO2 in Figure 9c for 5), the $\mathrm{d}_{x y} / \mathrm{d}_{x y}$ pathway may be considered to lead to the effective magnetic coupling through the di- $\mu$-oxo bridge in the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (II) complex. However, the $\mathrm{d}_{x y}$ orbital on the opposite Mn site would be nearly orthogonal to the bridging $\mathrm{p}_{\sigma}$ orbital, suggesting that the $\mathrm{d}_{x y} / \mathrm{d}_{x y}$ pathway is inactive for the antiferromagnetic interaction between $\mathrm{Mn}(\mathrm{II})$ sites and the magnetic interactions are thus weaker in $\mathbf{1}$ with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (II) than in $\mathbf{2}$ with $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ and $\mathbf{3}$ with $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$. Less orbital mixing of 3d orbitals between both Mn site is found in the complexes including the Mn (II) ion of $\mathbf{1}$ with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}$ (II), $\mathbf{4 a}$ and $\mathbf{4 b}$ with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{III})$, and 5 with $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$ (Appexdix C), indicating weak magnetic interactions also in these complexes.

The $\mathrm{d}_{x z} / \mathrm{d}_{x z}$ and $\mathrm{d}_{y z} / \mathrm{d}_{y z}$ pathways interacting with $\mathrm{p}_{\pi}$ orbital of the di- $\mu$-oxo bridge are common to 2 with $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ and 3 with $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})(\mathrm{SOMO} \pm 4$ and $\mathrm{SOMO} \pm 2$ in Figure 10b for 2 and $\mathrm{SOMO} \pm 2$ and $\mathrm{SOMO} \pm 3$ in Figure 10c for 3). The $d_{x z}$ and $d_{y z}$ orbitals respectively indicate the 6 and $6 \%$ contributions in 2 and the 7 and 8\% contributions in $\mathbf{3}$ from $\mu$-oxo ions, the 2 and $1 \%$ mixings in 2 and the 3 and $3 \%$ mixings in $\mathbf{3}$ between Mn sites, and the localizabilities of 91 and $90 \%$ in $\mathbf{2}$ and 84 and $84 \%$ in $\mathbf{3}$. However, in ref 30 the compositions of canonical orbitals were given as $\mathrm{Mn} / \mu$-oxo/ $\mathrm{Mn}=49: 39: 11$ for the $\mathrm{d}_{x z}$ orbital and 54:30:7 for the $\mathrm{d}_{y z}$ orbital in $\left[\mathrm{Mn}(\mathrm{III})_{2} \mathrm{O}_{2}{ }^{-}\right.$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{2+}$ and as $\mathrm{Mn} / \mu$-oxo/ $\mathrm{Mn}=35: 48: 15$ for the $\mathrm{d}_{x z}$ orbital and 43:27:15 for the $\mathrm{d}_{y z}$ orbital in $\left[\mathrm{Mn}(\mathrm{IV})_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{4+}$. Thus, it is found that the Mn-based orbitals are more delocalized in the canonical orbitals of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{9+}$ than in the magnetic orbitals of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$. The dependence of $\mathrm{d}_{x z} / \mathrm{d}_{x z}$ and $\mathrm{d}_{y z} /$ $\mathrm{d}_{y z}$ pathways in $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ on the geometry of the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core was also investigated in ref 30 . The authors' conclusions state that the $\mathrm{p}_{\pi}$ orbital of the $\mu$-oxo ion overlaps equally with the $\mathrm{d}_{x z}$ and $\mathrm{d}_{y z}$ orbitals at the point where the $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$ angle is exactly $90^{\circ}$. As the $\mathrm{O}-\mathrm{Mn}-\mathrm{O}$ angle becomes narrow, the interaction between the $\mathrm{p}_{\pi}$ orbital and the $\mathrm{d}_{y z}$ orbital is enhanced and the interaction between the $\mathrm{p}_{\pi}$ orbital and the $\mathrm{d}_{x z}$ orbital is reduced, respectively. The antiferromagnetic contribution from the $d_{y z} / d_{y z}$ pathway is, thus, enhanced at the optimized geometry, giving rise to a larger exchange coupling constant in $\mathrm{Mn}(\mathrm{IV})_{2}$ with $\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}=77.9^{\circ}$ than in $\mathrm{Mn}(\mathrm{III})_{2}$ with $\mathrm{O}-\mathrm{Mn}-$ (III) $-\mathrm{O}=84.8^{\circ}$. This trend would be true in the case of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2^{-}}\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$, although the difference in magnetic coupling between the $\mathrm{d}_{x z} / \mathrm{d}_{x z}$ and $\mathrm{d}_{y z} / \mathrm{d}_{y z}$ pathways may be small judging from the compositions of the $\mathrm{d}_{x z}$ and $\mathrm{d}_{y z}$ magnetic orbitals.

Regarding the remaining pathways of $\mathrm{d}_{x^{2}-y^{2} / \mathrm{d}_{x^{2}-y^{2}} \text { and } \mathrm{d}_{z} / \mathrm{d}_{z^{2}} \text {. }{ }^{2} \text {. }{ }^{2} \text {. }}$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ centers, it was shown in ref 30 that these symmetric pathways do not contribute because of the absence of a direct orbital overlap between Mn


Figure 10. Singly occupied magnetic orbitals of iso-valence complexes in the lowest-spin configuration: (a) $\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}(\mathbf{1})$, (b) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}(\mathbf{2})$, and (c) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{4+}$ (3). Isocontours correspond to 0.025 au.
sites, whereas the crossed pathway of $\mathrm{d}_{x^{2}-y^{2} / \mathrm{d}_{z}{ }^{2} \text { is possible with }}$ the compositions of canonical orbitals: $\operatorname{Mn}\left(\mathrm{d}_{x^{2}-y^{2}}\right) / \mu$-oxo/Mn$\left(\mathrm{d}_{z^{2}}\right)=62: 31: 4$ and $\operatorname{Mn}\left(\mathrm{d}_{z^{2}}{ }^{2} / \mu\right.$-oxo $/ \operatorname{Mn}\left(\mathrm{d}_{x^{2}-y^{2}}\right)=61: 5: 5$ in $[\mathrm{Mn}-$ $\left.(\mathrm{III})_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{2+}$ and $\mathrm{Mn}\left(\mathrm{d}_{\left.x^{2}-y^{2}\right) / \mu \text {-oxo } / \mathrm{Mn}\left(\mathrm{d}_{z^{2}}\right)=60: 25: 5 \text { in }}\right.$ $\left[\mathrm{Mn}(\mathrm{IV})_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{4+}$. The authors of ref 30 concluded that the $\mathrm{d}_{x^{2}-y^{2} / \mathrm{d}_{z^{2}} \text { pathway makes only minor contributions to the overall }}$ exchage coupling constant in the $\mathrm{Mn}($ III $)-\mathrm{Mn}($ III $)$ and $\mathrm{Mn}($ IV $)-$ $\mathrm{Mn}(\mathrm{IV})$ oxidation states, although it gives rise to additional antiferromagnetic (half filled $\mathrm{d}_{x^{2}-y^{2}}$ and $\mathrm{d}_{z^{2}}$ ) and ferromagnetic (half filled $\mathrm{d}_{x^{2}-y^{2}}$ and vacant $\mathrm{d}_{z^{2}}$ ) contributions in the $\mathrm{Mn}(\mathrm{III})_{2}$ and $\mathrm{Mn}(\mathrm{IV})_{2}$ dimers, respectively. However, the compositions of the magnetic orbitals of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ show that the $\mathrm{d}_{x^{2}-z^{2}}$ $\mathrm{d}_{x^{2}-z^{2}}$ and $\mathrm{d}_{y^{2}} / \mathrm{d}_{y^{2}}$ pathways in $2(\mathrm{SOMO} \pm 3$ and $\mathrm{SOMO} \pm 1$ in
 Figure 10(c) have small orbital mixings between Mn sites of 1 and $1 \%$ for $\mathrm{d}_{x^{2}-z^{2}}$ and $\mathrm{d}_{y^{2}}$ orbitals in 2 and $2 \%$ for the $\mathrm{d}_{x^{2}-y^{2}}$ orbital in $\mathbf{3}$, whereas the crossed pathway seems to be absent although the orbital mixings on the same Mn center between $\mathrm{d}_{x^{2}-z^{2}}$ and $\mathrm{d}_{y^{2}}$ in 2 and between $\mathrm{d}_{x^{2}-y^{2}}$ and $\mathrm{d}_{z^{2}}$ in $\mathbf{3}$ are found. Therefore, these symmetric pathways are also expected to
contribute secondly to the exchange coupling constant in addition to the dominant $\mathrm{d}_{x z} / \mathrm{d}_{x z}$ and $\mathrm{d}_{y z} / \mathrm{d}_{y z}$ pathways in the Mn (III) $-\mathrm{Mn}($ III $)$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ centers of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$, and it is suggested that the differences in contribution from the additional pathways may cause that $-J$ for $2>-J$ for $\mathbf{3}$ in contrast to that for $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$.

In the mixed-valence complex 6 with the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ center, the compositions of magnetic orbitals of $\mathrm{Mn}(\mathrm{III}) / \mu$-oxo/ $\mathrm{Mn}(\mathrm{IV})$ are given as follows: 84:2:2 and 2:13:74 for $\mathrm{d}_{x z}$ orbitals $\left(S O M O \pm 3\right.$ in Figure 11(d); 83:3:2 and 2:13:74 for $\mathrm{d}_{y z}$ orbitals (SOMO $\pm 1$ in Figure 11(d); 52:2:3 and 1:12:61 for the $\mathrm{d}_{y^{2}-z^{2}}$ orbital on $\mathrm{Mn}($ III $)$ and for the $\mathrm{d}_{x^{2}-y^{2}}$ orbital on $\mathrm{Mn}($ IV $)$ (SOMO $\pm 2$ in Figure 11d), respectively; 54:4:1 for the $\mathrm{d}_{x^{2}}$ orbital on Mn (III) (SOMO in Figure 9d). The SOMO and SOMO $\pm 2$ indicate the orbital mixings within the same Mn site. The contributions from the $\mathrm{d}_{x z} / \mathrm{d}_{x z}$ and $\mathrm{d}_{y z} / \mathrm{d}_{y z}$ pathways to the exchange coupling constant would also be dominant in the mixed-valence complex of $\left[\mathrm{Mn}(\mathrm{III}) \mathrm{Mn}(\mathrm{IV}) \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}$ as in the iso-valence complexes of $\left[\mathrm{Mn}(\mathrm{III})_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}$ and $[\mathrm{Mn}-$ $\left.(\mathrm{IV})_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{4+}$. The SOMO and $\mathrm{SOMO} \pm 2$ indicate the


Figure 11. Singly occupied magnetic orbitals of mixed-valence complexes in the lowest-spin configuration: (a) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}(4 a),(b)\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}(\mathbf{4 b})$, (c) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}(5)$, and (d) $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}(\mathbf{6})$. Isocontours correspond to 0.025 au
orbital mixings among the $\mathrm{d}_{x^{2}}\left(\mathrm{Mn}(\mathrm{III}), \mathrm{d}_{y^{2}-z^{2}} \mathrm{Mn}(\right.$ III $\left.)\right), \mathrm{d}_{x^{2}-y^{2}}$ $(\mathrm{Mn}(\mathrm{IV}))$, and $\mathrm{d}_{z^{2}}(\mathrm{MN}(\mathrm{IV}))$ components, suggesting the crossed pathway would be involved in these magnetic orbitals. The compositions of the canonical orbitals of $\left[\mathrm{Mn}(\right.$ III $) \mathrm{Mn}(\mathrm{IV}) \mathrm{O}_{2^{-}}$ $\left.\left(\mathrm{NH}_{3}\right)_{8}\right]^{3+}$ given in ref 30 again show the delocalized nature as $\operatorname{Mn}(\mathrm{III}) / \mu$-охо $/ \mathrm{Mn}(\mathrm{IV})=46: 34: 20$ and $8: 45: 46$ for the $\mathrm{d}_{x z}$ orbitals, 48:21:15 and 7:34:48 for the $\mathrm{d}_{\mathrm{yz}}$ orbitals, 57:20:14 $\left(\mathrm{d}_{z^{2}}\right)$ and $4\left(\mathrm{~d}_{z^{2}}\right): 28: 64$ for the $\mathrm{d}_{x^{2}-y^{2}}$ orbitals, and $48\left(\mathrm{~d}_{z^{2}}\right): 2: 12\left(\mathrm{~d}_{x^{2}-y^{2}}\right)$ for the unpaired orbital, indicating the crossed pathway of $\mathrm{d}_{x^{2}-y^{2}} /$ $\mathrm{d}_{z^{2}}$.

## IX. Summary

In the present study, we determined the stable geometries of di- $\mu$-oxo-bridged $\mathrm{Mn}_{2} \mathrm{O}_{2}$ complexes with $\mathrm{H}_{2} \mathrm{O}$ ligands [ $\mathrm{Mn}_{2} \mathrm{O}_{2}-$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q^{+}}(q=0-4)$, the iso-valence complexes in the Mn(II) -Mn (II), Mn (III) -Mn (III), and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}$ (IV) oxidation states, and the mixed-valence complexes in the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}-$ (III), $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ oxidation states, using all-electron DFT calculation of the broken-symmetry B3LYP method, and the geometrical features and electronic structures were investigated in relation to their changes depending on the oxidation state of $\mathrm{Mn}-\mathrm{Mn}$ center. The calculated results of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in this study were compared with those of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{9+}$ given in ref 30 .

The antiparallel spin-couplings are energetically more stable than the parallel spin-couplings for the oxidation states without the $\mathrm{Mn}(\mathrm{II})$ ion, whereas the energy differences between both spin alignments in the oxidation states with the $\mathrm{Mn}(\mathrm{II})$ ion are small. Although the $\mathrm{H}_{2} \mathrm{O}$ ligands of the $\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})$
sites are favorable to the octahedral positions, the $\mathrm{H}_{2} \mathrm{O}$ ligands of the $\mathrm{Mn}(\mathrm{II})$ site indicate the tendency to move from the octahedral positions, which may be caused by the instability of octahedral positions at the Mn (II) site induced from the electron occupations of antibonding orbitals in the axial and equatorial directions. The formation of hydrogen bonds are found between axial $\mathrm{H}_{2} \mathrm{O}$ ligands in the oxidation states except in $\mathrm{Mn}(\mathrm{II})-$ $\mathrm{Mn}(\mathrm{II})$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ and between $\mu$-oxo ions and $\mathrm{H}_{2} \mathrm{O}$ ligands in the oxidation states of $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II})$ and $\mathrm{Mn}(\mathrm{II})-$ Mn (III).

The optimized structures of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ are compatible with the reported structures of $\mathrm{Mn}_{2} \mathrm{O}_{2}$ complexes with various ligands, and the geometrical trends in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core depending on the oxidation state of complexes are indicated as $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ $>\mathrm{Mn}(\mathrm{III})-\mathrm{O} \approx \mathrm{Mn}(\mathrm{IV})-\mathrm{O}, \mathrm{O}-\mathrm{Mn}(\mathrm{II})-\mathrm{O}>\mathrm{O}-\mathrm{Mn}(\mathrm{III})-\mathrm{O}$ $>\mathrm{O}-\mathrm{Mn}(\mathrm{IV})-\mathrm{O}$ compared among iso-valence cores, and $\mathrm{O}-\mathrm{Mn}$ (lower) $-\mathrm{O}<\mathrm{O}-\mathrm{Mn}$ (higher) -O compared within the mixed-valence core. The comparisons of interatomic distances between the model complexes optimized in this study and the cubanelike Mn cluster of the OEC observed in the recent X-ray crystallographic study would suggest that the $\mathrm{Mn}(\mathrm{II})$ ion may not be contained in the $\mathrm{Mn}_{3} \mathrm{CaO}_{4}$ structure.

The charge and spin densities are evaluated by the Mulliken population analysis, and the electron populations in the Mn 3 d orbitals of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ seem to show correspondence with those of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$ for the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})-$ $\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ oxidation states. The singly occupied electron configurations of the $\mathrm{Mn}-\mathrm{Mn}$ center in each
oxidation state are elucidated by the natural orbital analysis, and it is confirmed that the one-electron oxidations from Mn(II) to Mn (III) and from Mn (III) to $\mathrm{Mn}(\mathrm{IV})$, respectively, remove the octahedral $\mathrm{e}_{\mathrm{g}}$ electrons in the equatorial $\left(\mathrm{d}_{x y}\right)$ and axial $\left(\mathrm{d}_{z^{2}}\right)$ orbitals.

The magnetic interactions between Mn sites through the $\mu$-oxo bridge are examined from the estimation of effective exchange integrals and superexchange pathways. The $J$ values of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}{ }^{-}\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III})$ and $\mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$ oxidation states seem to be consistent with the other experimental and theoretical values, although the $-J$ value is slightly larger for the $\mathrm{Mn}(\mathrm{III})_{2}$ dimer than for the $\mathrm{Mn}(\mathrm{IV})_{2}$ dimer in contrast with that of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{NH}_{3}\right)_{8}\right]^{q+}$. The $-J$ value of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2^{-}}\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the $\mathrm{Mn}(\mathrm{II})-\mathrm{Mn}(\mathrm{II})$ oxidation state becomes much
smaller than that in the higher oxidation states, indicating weak magnetic interactions through longer $\mathrm{Mn}(\mathrm{II})-\mathrm{O}$ separations. However, the $J$ value of the mixed-valence complex with the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ center reveals a large deviation from the reported $J$ values in other experimental and theoretical studies. Although the hydrogen bonds may affect the $J$ value, it is impossible to conclude the magnitude of the $J$ value exactly for $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{IV})$ oxidation state without uncertainty. The superexchange pathways of $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}-\right.$ $\left.\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the $\mathrm{Mn}(\mathrm{III})-\mathrm{Mn}(\mathrm{III}), \mathrm{Mn}(\mathrm{IV})-\mathrm{Mn}(\mathrm{IV})$, and $\mathrm{Mn}-$ (III) $-\mathrm{Mn}(\mathrm{IV})$ oxidation states based on the Mulliken compositions of singly occupied magnetic orbitals suggest that the symmetric pathways of $\mathrm{d}_{x z} / \mathrm{d}_{x z}$ and $\mathrm{d}_{y z} / \mathrm{d}_{y z}$ are dominant, and the remaining symmetric pathways would also contribute to the
$\underline{\text { TABLE A1: Optimized Interatomic Distances in the } \mathrm{Mn}_{2} \mathrm{O}_{2} \text { Core of Iso- and Mixed-Valence Complexes }\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}}$

| complex | $S$ | interatomic distance (angströms) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mn1-Mn2 | O3-O4 | Mn1-O3 | Mn1-O4 | Mn2-O3 | Mn2-O4 |
| 1 | 10/2 | 2.948 | 2.949 | 2.085 | 2.084 | 2.084 | 2.086 |
|  | 0/2 | 2.890 | 2.978 | 2.076 | 2.073 | 2.074 | 2.076 |
| 2 | 8/2 | 2.667 | 2.414 | 1.803 | 1.795 | 1.795 | 1.803 |
|  | 0/2 | 2.651 | 2.397 | 1.790 | 1.784 | 1.784 | 1.790 |
| 3 | 6/2 | 2.748 | 2.302 | 1.792 | 1.793 | 1.793 | 1.793 |
|  | 0/2 | 2.762 | 2.249 | 1.781 | 1.781 | 1.781 | 1.781 |
| 4a | 9/2 | 2.840 | 2.622 | 2.127 | 2.128 | 1.753 | 1.754 |
|  | 1/2 | 2.828 | 2.606 | 2.103 | 2.106 | 1.754 | 1.755 |
| 4b | 9/2 | 2.830 | 2.639 | 2.085 | 2.085 | 1.794 | 1.794 |
|  | 1/2 | 2.814 | 2.628 | 2.063 | 2.065 | 1.795 | 1.795 |
| 5 | 8/2 | 2.740 | 2.542 | 2.057 | 2.058 | 1.695 | 1.695 |
|  | 2/2 | 2.749 | 2.517 | 2.048 | 2.048 | 1.694 | 1.694 |
| 6 | 7/2 | 2.696 | 2.356 | 1.827 | 1.829 | 1.748 | 1.757 |
|  | 1/2 | 2.703 | 2.318 | 1.822 | 1.827 | 1.733 | 1.740 |

$\underline{\text { TABLE A2: Optimized Interatomic Angles in the } \mathbf{M n}_{2} \mathbf{O}_{2} \text { Core of Iso- and Mixed-Valence Complexes }\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathbf{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}}$

| complex | $S$ | interatomic angle (deg) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mn1-O3-Mn2 | Mn1-O4-Mn2 | $\mathrm{O} 3-\mathrm{Mn} 1-\mathrm{O} 4$ | $\mathrm{O} 3-\mathrm{Mn} 2-\mathrm{O} 4$ |
| 1 | 10/2 | 90.0 | 90.0 | 90.0 | 90.0 |
|  | 0/2 | 88.3 | 88.3 | 91.7 | 91.7 |
| 2 | 8/2 | 95.7 | 95.7 | 84.3 | 84.3 |
|  | $0 / 2$ | 95.8 | 95.7 | 84.2 | 84.2 |
| 3 | 6/2 | 100.0 | 100.0 | 79.9 | 79.9 |
|  | 0/2 | 101.7 | 101.7 | 78.3 | 78.3 |
| 4a | 9/2 | 93.6 | 93.5 | 76.0 | 96.8 |
|  | 1/2 | 93.9 | 93.7 | 76.5 | 95.9 |
| 4b | 9/2 | 93.4 | 93.4 | 78.5 | 94.7 |
|  | 1/2 | 93.4 | 93.4 | 79.0 | 94.0 |
| 5 | 8/2 | 93.3 | 93.3 | 76.3 | 97.2 |
|  | 2/2 | 94.0 | 94.0 | 75.8 | 96.0 |
| 6 | 7/2 | 97.9 | 97.5 | 80.2 | 84.5 |
|  | 1/2 | 98.9 | 98.5 | 78.9 | 83.7 |

TABLE A3: Optimized Interatomic Distances Between Mn Ions and $\mathbf{H}_{\mathbf{2}} \mathrm{O}$ Ligands of Iso- and Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{\mathbf{2}}\left(\mathbf{H}_{\mathbf{2}} \mathrm{O}\right)_{\mathbf{8}}\right]^{q+}$

| complex | $S$ | interatomic distance (angströms) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{Mn} 1-\mathrm{O}^{\text {a }}$ | $\mathrm{Mn1}-\mathrm{O6}^{\text {a }}$ | $\mathrm{Mn} 2-\mathrm{O} 7^{\text {a }}$ | $\mathrm{Mn} 2-\mathrm{O}^{\text {a }}$ | $\mathrm{Mn} 1-\mathrm{O} 9^{a}$ | $\mathrm{Mn} 1-\mathrm{O} 10^{a}$ | $\mathrm{Mn} 2-\mathrm{O} 11^{a}$ | $\mathrm{Mn} 2-\mathrm{O} 12^{\text {a }}$ |
| 1 | 10/2 | 2.261 | 2.265 | 2.261 | 2.264 | 2.238 | 2.238 | 2.239 | 2.238 |
|  | 0/2 | 2.268 | 2.269 | 2.266 | 2.270 | 2.244 | 2.245 | 2.244 | 2.244 |
| 2 | 8/2 | 2.414 | 2.255 | 2.255 | 2.413 | 2.092 | 2.089 | 2.090 | 2.093 |
|  | 0/2 | 2.406 | 2.250 | 2.251 | 2.405 | 2.100 | 2.098 | 2.098 | 2.100 |
| 3 | 6/2 | 1.969 | 1.969 | 1.969 | 1.969 | 2.051 | 2.051 | 2.051 | 2.051 |
|  | 0/2 | 1.968 | 1.968 | 1.968 | 1.968 | 2.049 | 2.049 | 2.049 | 2.049 |
| 4a | 9/2 | 2.200 | 2.188 | 2.816 | 3.003 | 2.226 | 2.231 | 2.095 | 2.094 |
|  | 1/2 | 2.196 | 2.184 | 2.779 | 2.959 | 2.241 | 2.245 | 2.096 | 2.095 |
| 4b | 9/2 | 2.362 | 2.361 | 2.289 | 2.288 | 2.159 | 2.159 | 2.119 | 2.119 |
|  | 1/2 | 2.384 | 2.379 | 2.284 | 2.284 | 2.165 | 2.165 | 2.122 | 2.122 |
| 5 | 8/2 | 2.350 | 2.350 | 2.010 | 2.010 | 2.180 | 2.180 | 2.163 | 2.164 |
|  | 2/2 | 2.334 | 2.333 | 2.008 | 2.008 | 2.186 | 2.187 | 2.160 | 2.161 |
| 6 | 7/2 | 2.380 | 2.381 | 1.967 | 1.968 | 2.048 | 2.042 | 2.066 | 2.076 |
|  | 1/2 | 2.356 | 2.357 | 1.969 | 1.969 | 2.058 | 2.051 | 2.066 | 2.072 |

[^1]exchange coupling constant, whereas the crossed pathway may be less important in the Mn dimers with $\mathrm{H}_{2} \mathrm{O}$ ligands than in the Mn dimers with $\mathrm{NH}_{3}$ ligands.

TABLE B1: Total and Relative Energies of the Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}$and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}$ in the Delocalized Highest-Spin State

| complex | $S$ | $\left\langle\hat{S}^{2}\right\rangle$ | total energy <br> $(\mathrm{au})$ | relative energy <br> $(\mathrm{kcal} / \mathrm{mol})$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{4}$ | $9 / 2$ | 24.90 | -3063.52301 | 0.00 |
|  | $9 / 2$ | 24.91 | -3063.50964 | 8.38 |
| $\mathbf{6}$ | $7 / 2$ | 16.05 | -3062.70282 | 0.00 |
|  | $7 / 2$ | 16.12 | -3062.63643 | 41.63 |

TABLE B2: Optimized Geometrical Parameters in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ Core of the Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)\right]^{+}$and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}$ in the Delocalized Highest-Spin State

| complex | $S$ | $\mathrm{Mn}-\mathrm{Mn}^{a}$ | $\mathrm{O}-\mathrm{O}^{a}$ | $\mathrm{Mn}-\mathrm{O}^{a}$ | $\mathrm{O}-\mathrm{Mn}-\mathrm{O}^{b}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{4}$ | $9 / 2$ | 2.615 | 2.708 | $1.881,1.883$ | 92.0 |
|  | $9 / 2$ | 2.833 | 2.554 | $1.905,1.909$ | 84.0 |
| $\mathbf{6}$ | $7 / 2$ | 2.683 | 2.374 | $1.790,1.792$ | 83.0 |
|  | $7 / 2$ | 2.794 | 2.291 | $1.805,1.808$ | 78.7 |

${ }^{a}$ Interatomic distances are given in angströms. ${ }^{b}$ Interatomic angles are given in degrees.

## Appendix A

The optimized geometrical parameters of iso- and mixedvalence complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{9+}$ in the most-stable highestand lowest-spin configurations are listed in Tables $\mathrm{A} 1-\mathrm{A} 3$ with Table A1 listing interatomic distances in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core, Table A2 listing interatomic angles in the $\mathrm{Mn}_{2} \mathrm{O}_{2}$ core, and Table A3 listing interatomic distances between Mn ions and $\mathrm{H}_{2} \mathrm{O}$ ligands.

## Appendix B

We obtained the delocalized highest-spin states of mixedvalence complexes $\mathbf{4}$ and $\mathbf{6}$ by applying geometry optimization with constraints for some of the geometrical parameters. The calculated results of the total energies, $\mathrm{Mn}_{2} \mathrm{O}_{2}$ structures, and $\mathrm{Mn}_{2} \mathrm{O}_{2}$ charge and spin densities are summarized in Tables B1B 3 , respectively.

## Appendix C

The singly occupied magnetic orbitals of the iso- and mixedvalence complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{q+}$ in the lowest-spin configuration are obtained from the singlet-coupled natural orbitals to recover the broken-symmetry picture. The Mulliken compositions of the magnetic orbitals of each complex are listed in

TABLE B3: Mulliken Charge and Spin Densities on Atoms in the $\mathbf{M n}_{2} \mathbf{O}_{\mathbf{2}}$ Core of the Mixed-Valence Complexes $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}$and $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}$ in the Delocalized Highest-Spin State

| complex | $S$ | charge density |  |  |  | spin density |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mn1 | Mn2 | O3 | O4 | Mn1 | Mn2 | O3 | O4 |
| 4 | 9/2 | 1.401 | 1.401 | $-1.058$ | -1.058 | 4.350 | 4.350 | 0.062 | 0.062 |
|  | 9/2 | 1.415 | 1.415 | -1.060 | -1.060 | 4.396 | 4.394 | 0.044 | 0.044 |
| 6 | 7/2 | 1.658 | 1.658 | -0.747 | -0.747 | 3.581 | 3.580 | -0.132 | -0.132 |
|  | 7/2 | 1.675 | 1.675 | -0.784 | -0.783 | 3.597 | 3.597 | -0.089 | -0.091 |

TABLE C1: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Iso-Valence Complex $\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}(\mathbf{1})$ in the Lowest-Spin Configuration

| magnetic orbital ${ }^{a}$ | W1 (eq) ${ }^{\text {b }}$ | W1 (ax) ${ }^{\text {b }}$ | $\mathrm{Mn} 1^{c}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | Mn2 ${ }^{\text {c }}$ |  |  |  |  |  | W2 (ax) ${ }^{e}$ | W2 (eq) ${ }^{\text {e }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}^{2}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{y^{2}-z^{2}}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{y^{2}-z^{2}}$ |  |  |
| $\mathrm{SOMO} \pm 1(+)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 97 | 0 | 0 | 0 | 1 | 1 |
| SOMO $\pm 1$ (-) | 1 | 1 | 0 | 0 | 97 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2(+)$ | 0 | 2 | 0 | 2 | 0 | 23 | 0 | 71 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2$ (-) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 0 | 23 | 0 | 71 | 2 | 0 |
| SOMO $\pm 3$ (+) | 2 | 2 | 0 | 92 | 0 | 0 | 0 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 3$ (-) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 92 | 0 | 0 | 0 | 3 | 2 | 2 |
| SOMO $\pm 4$ (+) | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 89 | 0 | 1 | 2 |
| SOMO $\pm 4$ (-) | 2 | 1 | 0 | 0 | 0 | 0 | 89 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| SOMO $\pm 5$ (+) | 0 | 1 | 2 | 1 | 0 | 72 | 0 | 23 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{SOMO} \pm 5(-)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 1 | 0 | 72 | 0 | 23 | 1 | 0 |

${ }^{a} \mathrm{SOMO} \pm m(+)$ and $\mathrm{SOMO} \pm m(-)$ correspond to plus and minus mixings of SOMO $+m$ and SOMO $-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn1, respectively. ${ }^{c}$ The first three and last two 3d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn1 and Mn2. ${ }^{e} \mathrm{~W} 2$ (ax) and W 2 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn2, respectively.

TABLE C2: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Iso-Valence Complex $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{\mathbf{2 +}}$ in the Lowest-Spin Configuration

| magnetic orbital ${ }^{a}$ | W1 (eq) ${ }^{b}$ | W1 (ax) ${ }^{\text {b }}$ | Mn1 ${ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | Mn2 ${ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{W} 2(\mathrm{ax})^{e}$ | $\mathrm{W} 2(\mathrm{eq})^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{x^{2}-z^{2}}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{x^{2}-z^{2}}$ |  |  |
| SOMO $\pm 1$ (+) | 1 | 2 | 1 | 0 | 0 | 85 | 0 | 4 | 5 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| SOMO $\pm 1$ (-) | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 5 | 1 | 0 | 0 | 85 | 0 | 4 | 2 | 1 |
| SOMO $\pm 2(+)$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 6 | 0 | 0 | 90 | 1 | 0 | 2 | 0 | 1 |
| SOMO $\pm 2$ (-) | 1 | 1 | 0 | 0 | 90 | 0 | 0 | 2 | 6 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 3(+)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 5 | 1 | 0 | 2 | 4 | 0 | 83 | 3 | 1 |
| SOMO $\pm 3$ (-) | 1 | 3 | 1 | 0 | 2 | 4 | 0 | 83 | 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| SOMO $\pm 4$ (+) | 1 | 0 | 0 | 91 | 0 | 0 | 0 | 0 | 6 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 4$ (-) | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 6 | 0 | 91 | 0 | 0 | 0 | 0 | 0 | 1 |

[^2]TABLE C3: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Iso-Valence Complex $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{4+}(3)$ in the Lowest-Spin Configuration

| magnetic orbital $^{a}$ | W1 (eq) ${ }^{\text {b }}$ | W1 (ax) ${ }^{\text {b }}$ | $\mathrm{Mn} 1{ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | Mn2 ${ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{W} 2(\mathrm{ax})^{e}$ | W2 (eq) ${ }^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}-y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{z^{2}}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}-y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{z^{2}}$ |  |  |
| $\mathrm{SOMO} \pm 1(+)$ | 1 | 0 | 0 | 0 | 0 | 64 | 0 | 26 | 6 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| SOMO $\pm 1$ (-) | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 6 | 0 | 0 | 0 | 64 | 0 | 26 | 0 | 1 |
| SOMO $\pm 2(+)$ | 2 | 2 | 0 | 84 | 0 | 0 | 0 | 2 | 7 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2$ (-) | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 7 | 0 | 84 | 0 | 0 | 0 | 2 | 2 | 2 |
| SOMO $\pm 3$ (+) | 0 | 1 | 0 | 0 | 3 | 0 | 0 | 0 | 8 | 0 | 0 | 84 | 0 | 0 | 0 | 4 | 2 |
| SOMO $\pm 3$ (-) | 2 | 4 | 0 | 0 | 84 | 0 | 0 | 0 | 8 | 0 | 0 | 3 | 0 | 0 | 0 | 1 | 0 |

${ }^{a} \mathrm{SOMO} \pm m(+)$ and $\mathrm{SOMO} \pm m(-)$ correspond to plus and minus mixings of $\mathrm{SOMO}+m$ and $\mathrm{SOMO}-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn 1 , respectively. ${ }^{c}$ The first three and last two 3d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn 1 and $\mathrm{Mn} 2 .{ }^{e} \mathrm{~W} 2$ (ax) and W 2 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn 2 , respectively.

TABLE C4: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Mixed-Valence Complex $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}$ (4a) in the Lowest-Spin Configuration

| magnetic orbital ${ }^{a}$ | W1 (eq) ${ }^{\text {b }}$ | W1 (ax) ${ }^{\text {b }}$ | Mn1 ${ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | $\mathrm{Mn} 2{ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{W} 2(\mathrm{ax})^{e}$ | W2 (eq) ${ }^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{x^{2}-z^{2}}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{y^{2}-z^{2}}$ |  |  |
| SOMO ${ }^{\text {f }}$ | 2 | 0 | 0 | 1 | 8 | 0 | 82 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| SOMO $\pm 1$ (+) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 6 | 0 | 0 | 38 | 0 | 51 | 1 | 0 |
| SOMO $\pm 1$ (-) | 1 | 0 | 0 | 0 | 0 | 92 | 0 | 4 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2(+)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 | 0 | 89 | 0 | 0 | 0 | 1 | 1 |
| SOMO $\pm 2(-)$ | 0 | 1 | 0 | 1 | 88 | 0 | 7 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 3$ (+) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 0 | 86 | 0 | 0 | 0 | 0 | 1 | 1 |
| SOMO $\pm 3$ (-) | 1 | 1 | 0 | 96 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 4$ (+) | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 10 | 2 | 0 | 0 | 46 | 0 | 38 | 0 | 1 |
| SOMO $\pm 4$ (-) | 1 | 4 | 0 | 0 | 1 | 4 | 0 | 89 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

${ }^{a} \mathrm{SOMO} \pm m(+)$ and $\mathrm{SOMO} \pm m(-)$ correspond to plus and minus mixings of $\mathrm{SOMO}+m$ and $\mathrm{SOMO}-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn 1 , respectively. ${ }^{c}$ The first three and last two 3d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn 1 and $\mathrm{Mn} 2 .{ }^{e} \mathrm{~W} 2$ (ax) and W 2 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn2, respectively. ${ }^{f}$ Compositions of the natural orbital are shown.

TABLE C5: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Mixed-Valence Complex $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{+}$ (4b) in the Lowest-Spin Configuration

| magnetic orbital $^{a}$ | W1 (eq) ${ }^{\text {b }}$ | W1 (ax) ${ }^{b}$ | Mn1 ${ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | $\mathrm{Mn}^{\text {c }}$ |  |  |  |  |  | $\mathrm{W} 2(\mathrm{ax})^{e}$ | $\mathrm{W} 2(\mathrm{eq})^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{x^{2}-z^{2}}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{y^{2}-z^{2}}$ |  |  |
| SOMO ${ }^{f}$ | 2 | 0 | 0 | 0 | 0 | 0 | 89 | 0 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| SOMO $\pm 1$ (+) | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 92 | 0 | 0 | 0 | 1 | 1 |
| SOMO $\pm 1$ (-) | 1 | 0 | 0 | 0 | 97 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2(+)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 1 | 1 | 0 | 25 | 0 | 64 | 4 | 1 |
| SOMO $\pm 2$ (-) | 1 | 1 | 0 | 1 | 0 | 95 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 3(+)$ | 2 | 2 | 1 | 87 | 0 | 1 | 0 | 6 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 3$ (-) | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 86 | 0 | 2 | 0 | 3 | 0 | 1 |
| SOMO $\pm 4(+)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 7 | 1 | 4 | 0 | 60 | 0 | 24 | 2 | 1 |
| SOMO $\pm 4$ (-) | 1 | 1 | 1 | 7 | 0 | 0 | 0 | 89 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

[^3]TABLE C6: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Mixed-Valence Complex $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{2+}$ (5) in the Lowest-Spin Configuration

| magnetic orbital ${ }^{a}$ | W 1 (eq) ${ }^{\text {b }}$ | W1 (ax) ${ }^{\text {b }}$ | $\mathrm{Mn}^{\text {c }}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | Mn2 ${ }^{\text {c }}$ |  |  |  |  |  | W2 (ax) ${ }^{\text {e }}$ | W2 (eq) ${ }^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}-y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{2}{ }^{2}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}-y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{2}{ }^{2}$ |  |  |
| SOMO1 ${ }^{f}$ | 2 | 0 | 0 | 0 | 0 | 46 | 35 | 12 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO2 ${ }^{f}$ | 2 | 0 | 0 | 0 | 0 | 31 | 54 | 8 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 1$ (+) | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 17 | 0 | 0 | 80 | 0 | 0 | 0 | 2 | 1 |
| SOMO $\pm 1$ (-) | 1 | 0 | 0 | 0 | 97 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2$ (+) | 1 | 3 | 1 | 3 | 0 | 19 | 0 | 73 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2$ (-) | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 14 | 0 | 2 | 0 | 71 | 0 | 10 | 0 | 1 |
| SOMO $\pm 3$ (+) | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 23 | 0 | 73 | 0 | 2 | 0 | 0 | 1 | 0 |
| SOMO $\pm 3$ (-) | 1 | 0 | 0 | 95 | 0 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

[^4]TABLE C7: Mulliken Compositions of the Singly Occupied Magnetic Orbitals of the Mixed-Valence Complex $\left[\mathrm{Mn}_{2} \mathrm{O}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8}\right]^{3+}$ (6) in the Lowest-Spin Configuration

| magnetic orbital ${ }^{a}$ | W1 (eq) ${ }^{b}$ | W1 (ax) ${ }^{\text {b }}$ | $\mathrm{Mn} 1{ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{OO}^{d}$ | Mn2 ${ }^{\text {c }}$ |  |  |  |  |  | $\mathrm{W} 2(\mathrm{ax})^{e}$ | $\mathrm{W} 2(\mathrm{eq})^{e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{y^{2}-z^{2}}$ |  | sp | $\mathrm{d}_{x z}$ | $\mathrm{d}_{y z}$ | $\mathrm{d}_{x^{2}-y^{2}}$ | $\mathrm{d}_{x y}$ | $\mathrm{d}_{z^{2}}$ |  |  |
| SOMO ${ }^{\text {f }}$ | 1 | 1 | 0 | 0 | 0 | 54 | 0 | 40 | 4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| SOMO $\pm 1$ (+) | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 13 | 0 | 9 | 74 | 0 | 0 | 0 | 2 | 1 |
| SOMO $\pm 1$ (-) | 1 | 0 | 0 | 10 | 83 | 0 | 0 | 0 | 3 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| SOMO $\pm 2(+)$ | 1 | 4 | 1 | 0 | 0 | 37 | 0 | 52 | 2 | 0 | 0 | 0 | 2 | 0 | 1 | 0 | 0 |
| SOMO $\pm 2$ (-) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 12 | 0 | 0 | 0 | 61 | 0 | 23 | 0 | 1 |
| SOMO $\pm 3$ (+) | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 13 | 0 | 74 | 8 | 0 | 0 | 0 | 1 | 1 |
| SOMO $\pm 3$ (-) | 2 | 0 | 0 | 84 | 10 | 0 | 0 | 0 | 2 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |

${ }^{a} \mathrm{SOMO} \pm m(+)$ and $\mathrm{SOMO} \pm m(-)$ correspond to plus and minus mixings of $\mathrm{SOMO}+m$ and $\mathrm{SOMO}-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn 1 , respectively. ${ }^{c}$ The first three and last two 3d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn 1 and $\mathrm{Mn} 2 .{ }^{e} \mathrm{~W} 2$ (ax) and W 2 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn2, respectively. ${ }^{f}$ Compositions of the natural orbital are shown.

Tables C1-C7 as Table C1 for 1, Table C2 for 2, Table C3 for $\mathbf{3}$, Table C 4 for $\mathbf{4 a}$, Table C5 for $\mathbf{4 b}$, Table C6 for $\mathbf{5}$, and Table C7 for 6.

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[^1]:    ${ }^{a} \mathrm{O} 5-\mathrm{O} 12$ indicate O atoms in $\mathrm{H}_{2} \mathrm{O}$ molecules shown in Figure 3, respectively.

[^2]:    ${ }^{a} \mathrm{SOMO} \pm m(+)$ and $\mathrm{SOMO} \pm m(-)$ correspond to plus and minus mixings of SOMO $+m$ and $\mathrm{SOMO}-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn 1 , respectively. ${ }^{c}$ The first three and last two 3 d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn 1 and $\mathrm{Mn} 2 .{ }^{e} \mathrm{~W} 2$ (ax) and W 2 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn2, respectively.

[^3]:    ${ }^{a}$ SOMO $\pm m(+)$ and SOMO $\pm m(-)$ correspond to plus and minus mixings of SOMO $+m$ and SOMO $-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn1, respectively. ${ }^{c}$ The first three and last two 3d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn1 and Mn2. ${ }^{e} \mathrm{~W} 2(\mathrm{ax})$ and $\mathrm{W} 2(e q)$ indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn2, respectively. ${ }^{f}$ Compositions of the natural orbital are shown.

[^4]:    ${ }^{a} \mathrm{SOMO} \pm m(+)$ and SOMO $\pm m(-)$ correspond to plus and minus mixings of SOMO $+m$ and SOMO $-m$, respectively. ${ }^{b} \mathrm{~W} 1$ (ax) and W 1 (eq) indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn 1 , respectively. ${ }^{c}$ The first three and last two 3d orbitals correspond to the octahedral $\mathrm{t}_{2 \mathrm{~g}}$ and $\mathrm{e}_{\mathrm{g}}$ orbitals, respectively. ${ }^{d} \mathrm{OO}$ indicates a di- $\mu$-oxo bridge between Mn1 and Mn2. ${ }^{e} \mathrm{~W} 2(\mathrm{ax})$ and $\mathrm{W} 2(\mathrm{eq})$ indicate axial and equatorial $\mathrm{H}_{2} \mathrm{O}$ molecules coordinated to Mn2, respectively. ${ }^{f}$ Compositions of the natural orbital are shown.

